$$\sqrt[4]{624} = \sqrt[4]{625+(-1)} \qquad (x^{1/4})' = \frac{1}{4}x^{-3/4}$$

$$\approx \sqrt[4]{625} + \left(\sqrt[4]{x}\right)'\Big|_{x=625} \cdot (-1)$$

$$\sqrt[4]{624} \approx 5 + \frac{-1}{4\left(\sqrt[4]{625}\right)^3} = 5 - \frac{1}{500}$$

$$= \frac{2499}{500}$$

$$x^{m/n} = \left(\sqrt[n]{x}\right)^m$$

$$= \sqrt[n]{x^m}$$

4.1 Antiderivatives

$$F(x) = 5x^4$$

$$f(x) = x^5 \qquad [f(x)]' = 5x^4$$

particular solution

General solution: $x^5 + C$

$$y = F(x)$$

$$\frac{dy}{dx} = f(x)$$

$$dy = f(x)\,dx$$

$$\int dy = \int f(x)\,dx$$

$$y = \int f(x)\,dx = F(x) + C$$

antiderivative = indefinite integral

18. $\int (4x^3 + 6x^2 - 1)\,dx$

$= \boxed{x^4 + 2x^3 - x + C}$

24. $\int (\sqrt[4]{x^3} + 1)\,dx = \int (x^{3/4} + 1)\,dx$

$= \boxed{\frac{4}{7}x^{7/4} + x + C}$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1$

$x^{-1} = \frac{1}{x}$

$\int x^6\,dx = \frac{x^7}{7} + C$

$\frac{x^m}{x^n} = x^{m-n}$

28. $\int \frac{x^2 + 2x - 3}{x^4}\,dx = \int (x^{-2} + 2x^{-3} - 3x^{-4})\,dx$

" $\int \left(\frac{x^2}{x^4} + \frac{2x}{x^4} - \frac{3}{x^4}\right)dx$ "

$= \frac{x^{-2+1}}{-1} + \frac{2x^{-3+1}}{-2} - \frac{3x^{-4+1}}{-3} + C$

$= -x^{-1} - x^{-2} + x^{-3} + C$

$= \boxed{-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C}$

38. $\int (\theta^2 + \sec^2\theta)\,d\theta = \boxed{\frac{1}{3}\theta^3 + \tan\theta + C}$

42. $\int \frac{\cos x}{1-\cos^2 x}\,dx = \int \frac{\cos x}{\sin^2 x}\,dx$

$= \int \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}\,dx = \int \csc x \cot x\,dx$

$= \boxed{-\csc x + C}$

40. $\int \sec y(\tan y - \sec y)\,dy$

$= \int (\sec y \tan y - \sec^2 y)\,dy$

$= \boxed{\sec y - \tan y + C}$

Find a general solution and a particular solution to the differential equation.

56. $g'(x) = 6x^2$, $g(0) = -1$

$y' = 6x^2$

$\frac{dy}{dx} = 6x^2$

$\int dy = \int 6x^2\,dx$

$y = 2x^3 + C$ ← general solution

$g(0) = -1$ $\quad -1 = 2(0)^3 + C$

$-1 = C$

particular solution: $\boxed{y = 2x^3 - 1}$

58. $f'(s) = 6s - 8s^3$, $f(2) = 3$

$\frac{df}{ds} = 6s - 8s^3$

$df = (6s - 8s^3)dx$

$f = 3s^2 - 2s^4 + C$ ← general solution

$3 = 3(2)^2 - 2(2)^4 + C$

$3 = 12 - 32 + C$

$23 = C$

particular solution: $\boxed{f(s) = 3s^2 - 2s^4 + 23}$

62. $f''(x) = \sin x$, $f'(0) = 1$, $f(0) = 6$

$f'(x) = -\cos x + C_1$

$f(x) = -\sin x + C_1 x + C_2$ ← general solution

$1 = -\cos 0 + C_1$

$1 = -1 + C_1$

$2 = C_1$

$6 = -\sin 0 + 2(0) + C_2$

$6 = C_2$

particular solution: $\boxed{f(x) = -\sin x + 2x + 6}$

4.1

15-31 odd ← antiderivatives

37-41 odd ← diff eq's