Summation Formulas

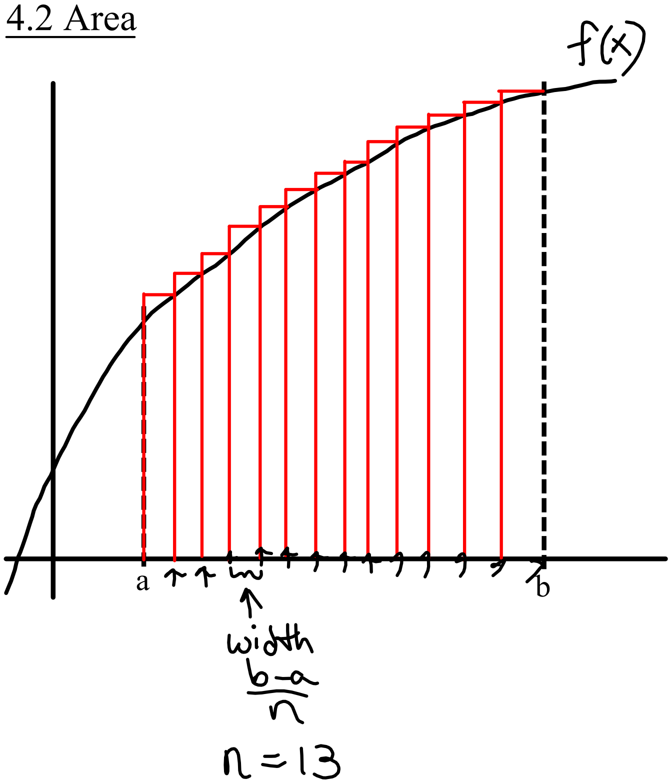
1. $\sum_{i=1}^{n} c = nc$

2. $\sum_{i=1}^{n} i = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\ldots+n^3 = \frac{n^2(n+1)^2}{4}$

4.2 Area



Divide the interval [a,b] into n equal subintervals, each of width (b-a)/n.

Here, the height of a rectangle is determined by the right endpoint of each subinterval; this is called an upper sum.

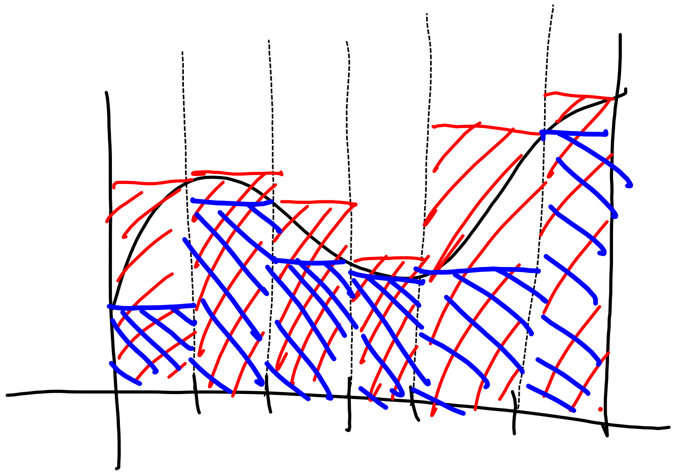
The area of each rectangle will be:

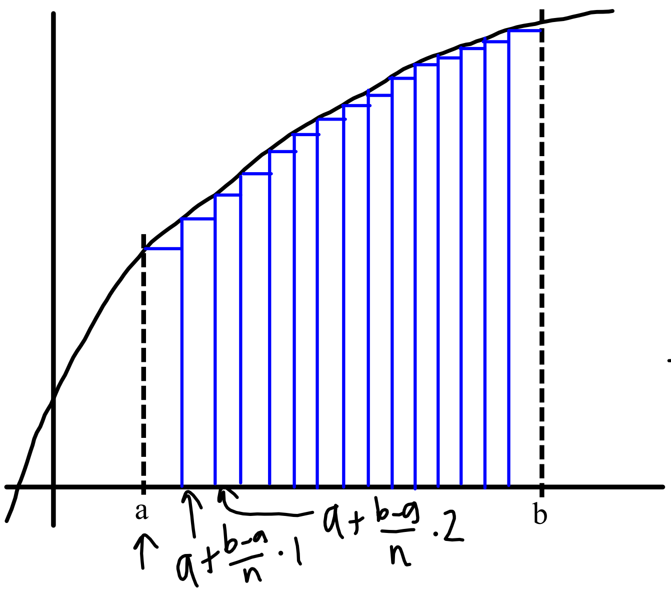
$$\frac{b-a}{n} \cdot f\left(a+\frac{b-a}{n}i\right)$$

Total area:

$$\sum_{i=1}^{13} \frac{b-a}{n} \cdot f\left(a+\frac{b-a}{n}i\right)$$

for right-hand sum





Here, the height of a rectangle is determined by the left endpoint of each subinterval; this is called a lower sum.

The area of each rectangle will be:

$$\frac{b-a}{n}\cdot f\left(a+\frac{b-a}{n}(i-1)\right)$$

Total area

$$\sum_{i=1}^{n} \frac{b-a}{n} f\left(a+\frac{b-a}{n}\cdot(i-1)\right)$$

using left-hand endpoint

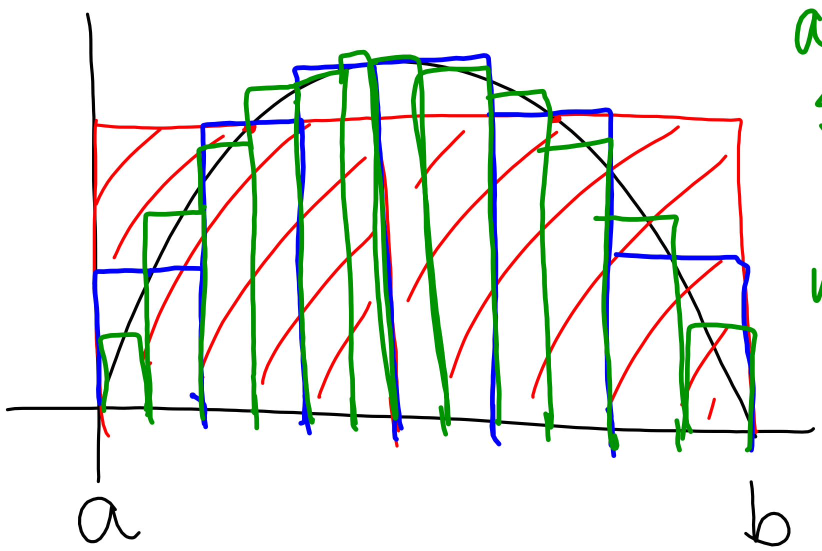
Lower sum: $s(n) = \sum_{i=1}^{n} f(m_i)\Delta x$

Upper sum: $S(n) = \sum_{i=1}^{n} f(M_i)\Delta x$

$f(m_i)$ = minimum function value in an interval

$f(M_i)$ = Maximum function value in an interval

$\Delta x = \frac{b-a}{n}$ $s(n) \leq S(n)$



as # of subdivisions $n \to \infty$ we get a better approximation for area under curve

Area of the region bounded by the graph of f, the x-axis, & the lines x=a & x=b is

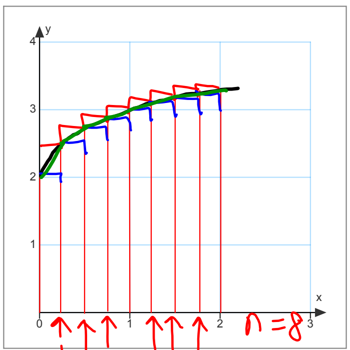
$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \quad , x_{i-1} \le c_i \le x_i$$

where $\Delta x = \dfrac{b-a}{n}$.

28. $y = \sqrt{x} + 2$

Use upper sum and lower sum to approximate the area under the curve.

width $\Delta x = \dfrac{b-a}{n} = \dfrac{2-0}{8} = \dfrac{1}{4}$



$n = 8$

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{4}$ $\frac{3}{2}$ $\frac{7}{4}$

upper sum:

$$\frac{1}{4}\left(\sqrt{\tfrac{1}{4}}+2 + \sqrt{\tfrac{1}{2}}+2 + \sqrt{\tfrac{3}{4}}+2 + \sqrt{1}+2 + \sqrt{\tfrac{5}{4}}+2 + \sqrt{\tfrac{3}{2}}+2 + \sqrt{\tfrac{7}{4}}+2 + \sqrt{2}+2\right)$$

≈ 6.038

lower sum:

$$\frac{1}{4}\left(\sqrt{0}+2 + \sqrt{\tfrac{1}{4}}+2 + \sqrt{\tfrac{1}{2}}+2 + \sqrt{\tfrac{3}{4}}+2 + \sqrt{1}+2 + \sqrt{\tfrac{5}{4}}+2 + \sqrt{\tfrac{3}{2}}+2 + \sqrt{\tfrac{7}{4}}+2\right)$$

≈ 5.6847

upper:

$$\sum_{i=1}^{8} \frac{1}{4}\left(\sqrt{i/4}+2\right)$$

lower:

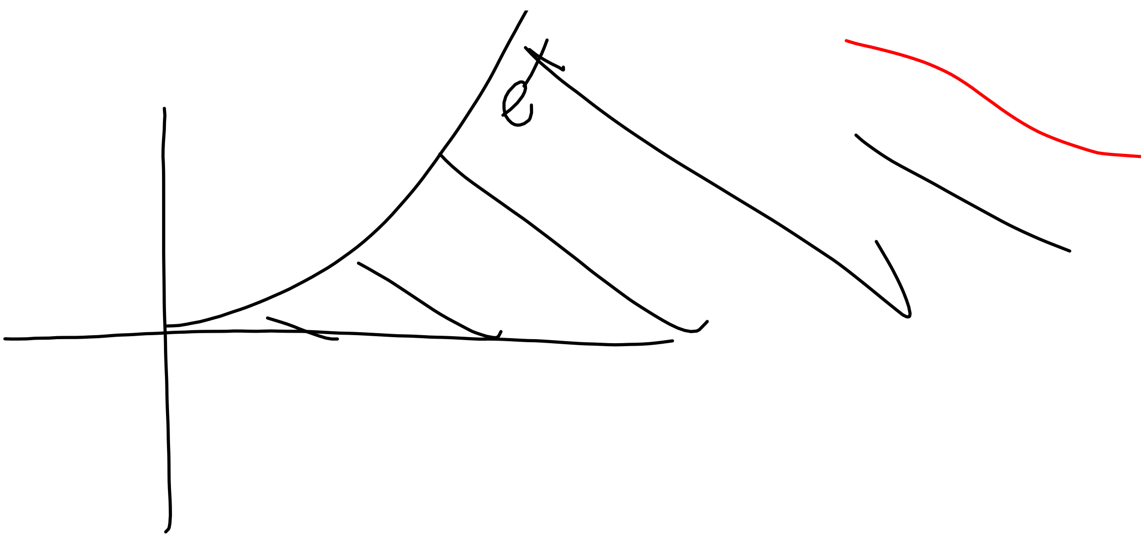
$$\sum_{i=1}^{8} \frac{1}{4}\left(\sqrt{\tfrac{i-1}{4}}+2\right)$$

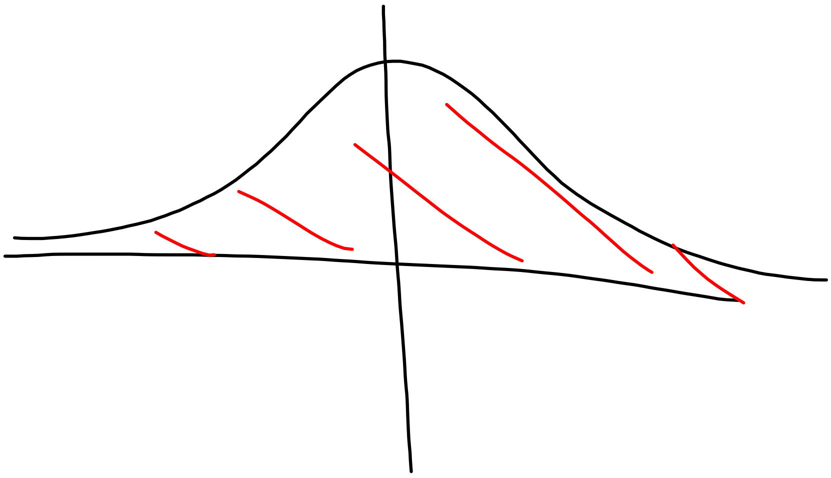
trapezoidal sum is average of upper & lower sums

$$\lim_{n\to\infty} S(n)$$

32. $S(n) = \dfrac{64}{n^3}\left[\dfrac{n(n+1)(2n+1)}{6}\right]$

$$\lim_{n\to\infty} S(n) = \lim_{n\to\infty} \frac{64\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{2}\cancel{n}}{\cancel{n^3}\cdot\cancel{6}3} = \boxed{\frac{64}{3}}$$





rewrite without summation notation

36. $\sum_{j=1}^{n} \frac{4j+3}{n^2} = \frac{4 \cdot 1+3}{n^2} + \frac{4 \cdot 2+3}{n^2} + \cdots + \frac{4 \cdot n+3}{n^2}$

$= \frac{1}{n^2} \sum_{j=1}^{n} (4j+3) = \frac{1}{n^2} \left(\sum_{j=1}^{n} 4j + \sum_{j=1}^{n} 3 \right)$

$= \frac{1}{n^2} \left(4 \sum_{j=1}^{n} j + \sum_{j=1}^{n} 3 \right)$

$= \frac{1}{n^2} \left[4 \cdot \frac{n(n+1)}{2} + 3n \right] = \frac{4n(n+1)}{2n^2} + \frac{3n}{n^2}$

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