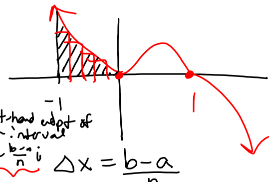
56. $y = x^2 - x^3 \quad [-1, 0]$

$y = x^2(1-x)$



right-hand endpt of i^{th} interval

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i \qquad \underbrace{a + \frac{b-a}{n} i} \qquad \Delta x = \frac{b-a}{n}$$

$$c_i = -1 + \frac{i}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \cdot \left[\left(\frac{i}{n}-1\right)^2 - \left(\frac{i}{n}-1\right)^3\right] \qquad = \frac{0-(-1)}{n} = \frac{1}{n}$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left[\frac{i^2}{n^2} - \frac{2i}{n} + 1 - \left(\frac{i^3}{n^3} - \frac{3i^2}{n^2} + \frac{3i}{n} - 1\right)\right]$$

$(a-b)^2 = a^2 - 2ab + b^2 \qquad (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left[2 - \frac{5i}{n} + \frac{4i^2}{n^2} - \frac{i^3}{n^3}\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\sum_{i=1}^{n} 1 - \frac{5}{n^2}\sum_{i=1}^{n} i + \frac{4}{n^3}\sum_{i=1}^{n} i^2 - \frac{1}{n^4}\sum_{i=1}^{n} i^3\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\cdot n - \frac{5}{n^2}\cdot\frac{n(n+1)}{2} + \frac{4}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{1}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right]$$

$$= 2 - \frac{5}{2} + \frac{4}{3} - \frac{1}{4} =$$

$$= \frac{24}{12} - \frac{30}{12} + \frac{16}{12} - \frac{3}{12} = \boxed{\frac{7}{12}}$$

4.3 Riemann Sums & Definite Integrals

$$\sum_{i=1}^{n} f(c_i)\Delta x_i \quad , \quad x_{i-1} \le c_i \le x_i ,$$

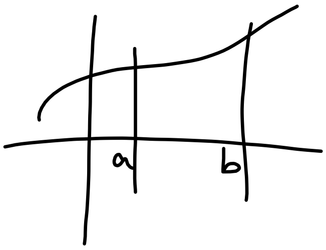
where c_i is any point in the i^{th} subinterval ; $a = x_0 < x_1 < x_2 < \ldots < x_n = b$ is called the <u>Riemann Sum</u> of f.

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x)\,dx$$

called the <u>definite integral</u> of f from a to b.

<u>Properties</u>

If $f(a)$ is defined,



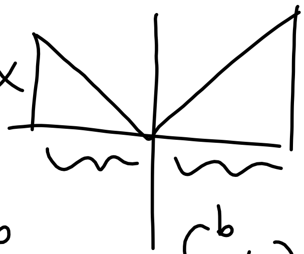
$$\int_a^a f(x)\,dx = 0$$

If f is integrable on $[a,b]$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$



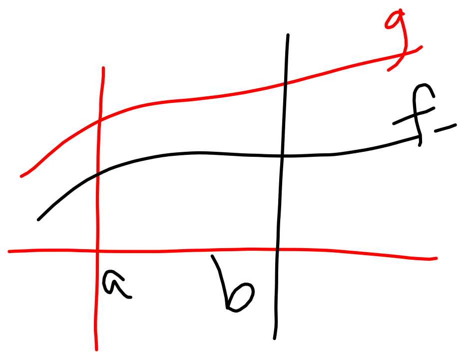
$$\int_a^b \left[f(x) \pm g(x)\right]dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

If $f(x) \geq 0$,

$$\int_a^b f(x)\,dx \geq 0$$

If $f(x) \leq g(x)$

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

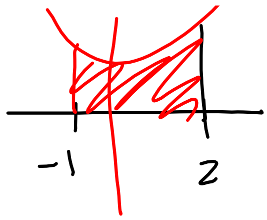


4.4 The Fundamental Theorem of Calculus

If f is cts on $[a,b]$ and F is antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

8. $\displaystyle\int_{-1}^{2} (3x^2+2)\,dx = x^3+2x \Big|_{x=-1}^{2}$



$$= \left[2^3+2(2)\right] - \left[(-1)^3 + 2(-1)\right]$$

$$= 8+4+1+2 = \boxed{15}$$

10. $\int_1^3 (3x^2+5x-4)\,dx$

$= x^3+\frac{5}{2}x^2-4x\Big|_{x=1}^{3}$

$= \left[3^3+\frac{5}{2}(3)^2-4(3)\right] - \left[1^3+\frac{5}{2}(1)^2-4(1)\right] =$

$= 27+\frac{45}{2}-12-1-\frac{5}{2}+4$

$= 27+20-12-1+4$

$= \boxed{38}$