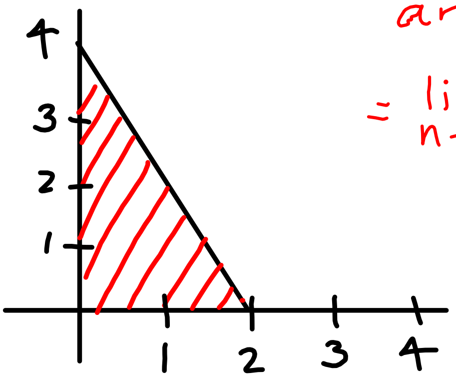
4.4 The Fundamental Theorem of Calculus

If f is cts on $[a,b]$ and F is antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

14. $f(x) = 4 - 2x$



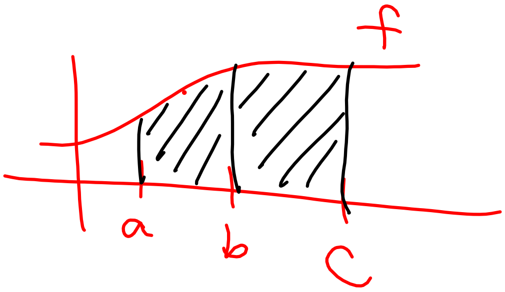
$$\text{area} = \tfrac{1}{2}(2)(4) = 4$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[4 - 2\left(\frac{2i}{n}\right)\right]$$

$$= \int_0^2 (4-2x)\,dx = 4x - x^2 \Big|_{x=0}^{x=2}$$

$$= 4(2) - 2^2 - \left(4(0) - 0^2\right) = 4$$

$$\int_a^c f(x) = dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$



$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

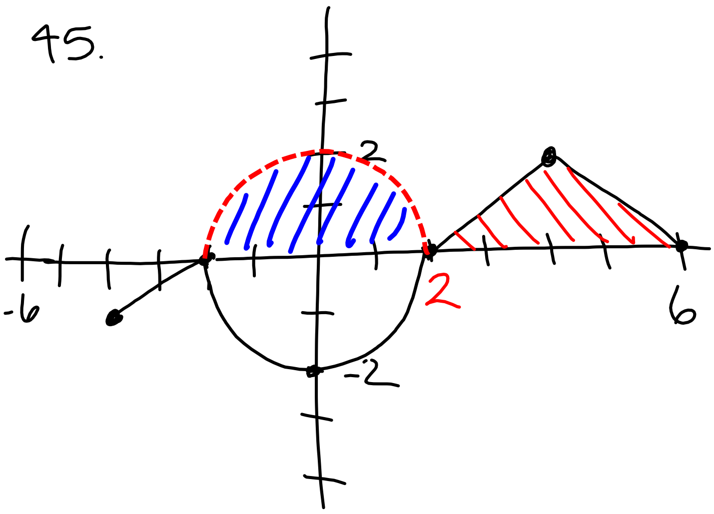
40. $\int_2^4 x^3\,dx = 60$; $\int_2^4 x\,dx = 6$; $\int_2^4 dx = 2$

$$\int_2^4 (6+2x-x^3)\,dx = \int_2^4 6\,dx + \int_2^4 2x\,dx - \int_2^4 x^3\,dx$$

$$= 6\int_2^4 dx + 2\int_2^4 x\,dx - \int_2^4 x^3\,dx$$

$$= 6\cdot 2 + 2\cdot 6 - 60 = \boxed{-36}$$

45.



b) $\int_2^6 f(x)\,dx$

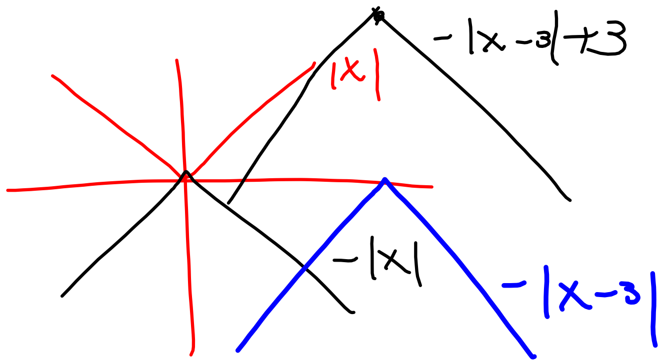
$= \frac{1}{2}(4)(2) = \boxed{4}$

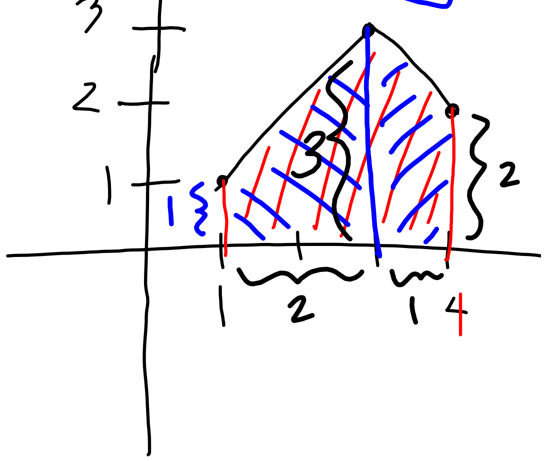
$\int_{-2}^{2} |f(x)|\,dx$

$\frac{1}{2}\pi(2)^2 = \boxed{2\pi}$

24. $\int_1^4 (3-|x-3|)\,dx = \frac{1}{2}(1+3)(2) + \frac{1}{2}(3+2)(1)$

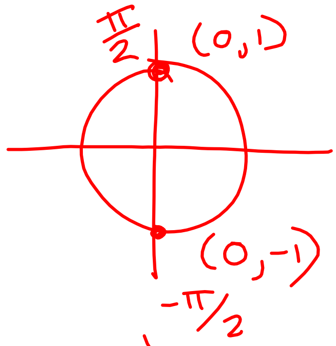
$4 + \frac{5}{2} = \boxed{\frac{13}{2}}$





32. $\displaystyle\int_{-\pi/2}^{\pi/2} (2t + \cos t)\,dt$

$= t^2 + \sin t \Big|_{t=-\pi/2}^{\pi/2}$



$= \cancel{\left(\frac{\pi}{2}\right)^2} + \sin\frac{\pi}{2} - \left(\cancel{\left(\frac{-\pi}{2}\right)^2} + \sin\left(\frac{-\pi}{2}\right)\right)$

$= 1 - (-1) = \boxed{2}$

<u>4.3</u>

$\int_a^c = \int_a^b + \int_b^c$ $\qquad$ $\int_a^b = -\int_b^a$

44. Given $\displaystyle\int_{-1}^{1} f(x)\,dx = 0$ & $\displaystyle\int_0^1 f(x)\,dx = 5$

$\int_{-1}^{1} = \int_{-1}^{0} + \int_0^1$

(a) $\displaystyle\int_{-1}^{0} f(x)\,dx = \int_{-1}^{1} - \int_0^1 = \boxed{-5}$

(b) $\displaystyle\int_0^1 f(x)\,dx - \int_{-1}^{0} f(x)\,dx = \boxed{-10}$

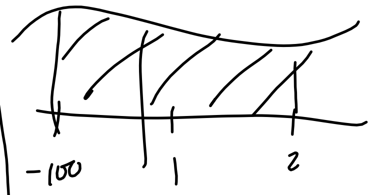
(c) $\displaystyle\int_{-1}^{1} 3f(x)\,dx = 3\cdot 0 = \boxed{0}$

(d) $\displaystyle\int_0^1 3f(x)\,dx = 3\cdot 5 = \boxed{15}$

(e) $\displaystyle\int_1^0 f(x)\,dx = \boxed{-5}$

$\displaystyle\int_1^2 = \int_1^{-100} + \int_{-100}^{2}$

$\displaystyle = -\int_{-100}^{1} + \int_{-100}^{2}$



$$\int_{-1}^{1} f(x)\,dx = 0 \quad ; \quad \int_{0}^{1} f(x) = 5$$

$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$

$$\int_{-1}^{1} f(x)\,dx - \int_{0}^{1} f(x) = \int_{-1}^{0} f(x)\,dx$$

$$0 - 5 = \boxed{-5}$$

$$\int_{0}^{1} f(x)\,dx - \int_{-1}^{0} f(x)\,dx$$

$$5 - (-5)$$

$$= \boxed{10}$$

$$\int_{-1}^{1} = 0$$

$$\int_{0}^{1} = 5$$

$$\int_{-1}^{0} = -5$$

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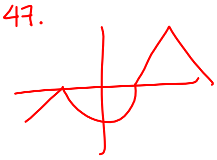
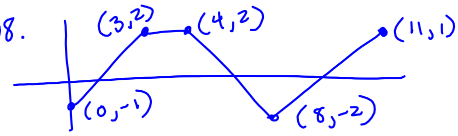
47.



(a) $\int_0^2 f(x)\,dx$
(b) $\int_2^6 f(x)\,dx$
(c) $\int_{-4}^2 f(x)\,dx$
(d) $\int_{-4}^6 f(x)\,dx$
(e) $\int_{-4}^6 |f(x)|\,dx$
(f) $\int_{-4}^6 [f(x)+2]\,dx$

48.



a) $\int_0^1 -f(x)\,dx$
b) $\int_3^4 3f(x)\,dx$
c) $\int_0^7 f(x)\,dx$
d) $\int_5^{11} f(x)\,dx$
e) $\int_0^{11} f(x)\,dx$ f) $\int_4^{10} f(x)\,dx$

49. f is continuous on $[5,5]$ & $\int_0^5 f(x)\,dx = 4$
Evaluate:

a) $\int_0^5 [f(x)+2]\,dx$ (b) $\int_{-2}^3 f(x+2)\,dx$

(c) $\int_{-5}^5 f(x)\,dx$ (f is even) (d) $\int_{-5}^5 f(x)\,dx$ (f is odd)

11.

$$\frac{dP}{dt} = k\sqrt[3]{t}$$

$$\int dP = \int k t^{1/3}\,dt$$

$$P = \qquad + C$$