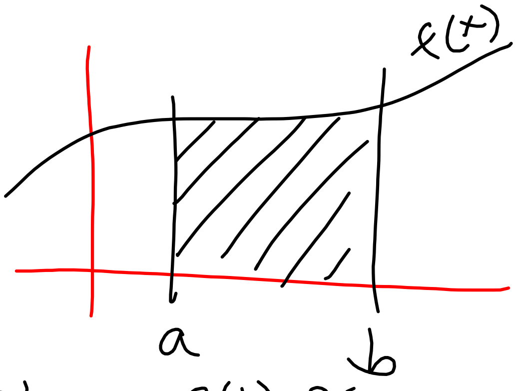

$\int_a^b f(x)\,dx =$ area under the curve

$\int_a^b f(x)\,dx = F(b) - F(a)$

$f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$(b-a)\,f'(c) = f(b) - f(a) \Rightarrow (b-a)\,f(c) = F(b) - F(a)$

$f(c) = \dfrac{1}{b-a}\int_a^b f(x)\,dx \Leftarrow (b-a)\underbrace{f(c)}_{\text{average function value}} = \int_a^b f(x)\,dx$

4.5 Integration by Substitution

12. $\int x^2(x^3+5)^4\,dx = \int (x^3+5)^4 \cdot x^2\,dx$

Let $u = x^3 + 5$

$\dfrac{du}{3} = \dfrac{\cancel{3}x^2\,dx}{\cancel{3}}$

$= \int u^4\left(\frac{1}{3}du\right)$

$= \frac{1}{3}\cdot\frac{1}{5}u^5 + C$

$= \boxed{\frac{1}{15}(x^3+5)^5 + C}$

22. $\displaystyle\int \frac{x^2}{(16-x^3)^2}\,dx = \int \frac{1}{u^2}\cdot\left(\frac{-1}{3}\right)du$

$u = 16 - x^3$

$\dfrac{du}{-3} = \dfrac{-3x^2\,dx}{-3}$

$-\frac{1}{3}du = x^2\,dx$

$= \int -\frac{1}{3}u^{-2}\,du$

$= \frac{1}{3}u^{-1} + C$

$= \boxed{\dfrac{1}{3(16-x^3)} + C}$

4.5

50. $\int \sqrt{\tan x}\,\sec^2 x\,dx = \int \sqrt{u}\,du$

$u = \tan x$

$du = \sec^2 x\,dx$

$= \int u^{1/2}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{3}(\tan x)^{3/2} + C}$

54. $\int \csc^2\left(\frac{x}{2}\right)dx = \int \csc^2 u \cdot 2\,du$

$u = \frac{x}{2} = \frac{1}{2}x$

$du = \frac{1}{2}dx$

$2\,du = dx$

$= \int 2\cdot\csc^2 u\,du$

$= -2\cot u + C$

$= \boxed{-2\cot\left(\frac{x}{2}\right) + C}$

52. $\displaystyle\int \frac{\sin x}{\cos^3 x}\,dx = \int \frac{-du}{u^3} = \int -u^{-3}\,du$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$= \frac{1}{2}u^{-2} + c$

$= \dfrac{1}{2\cos^2 x} + c$

$= \boxed{\frac{1}{2}\sec^2 x + c}$

58. $\displaystyle\int x\sqrt{2x+1}\,dx = \int \frac{u-1}{2}\cdot\sqrt{u}\cdot\frac{1}{2}du$

$u = 2x+1 \longrightarrow \frac{u-1}{2} = x$

$du = 2dx$

$\frac{1}{2}du = dx$

$= \displaystyle\int \frac{1}{2}(u-1)\cdot u^{1/2}\cdot\frac{1}{2}du$

$= \displaystyle\frac{1}{4}\int (u-1)u^{1/2}\,du$

$= \displaystyle\frac{1}{4}\int \left(u^{3/2} - u^{1/2}\right)du$

$= \frac{1}{4}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + c$

$= \frac{2}{20}(2x+1)^{5/2} - \frac{2}{12}(2x+1)^{3/2} + c$

$= \boxed{\frac{1}{10}\sqrt{(2x+1)^5} - \frac{1}{6}\sqrt{(2x+1)^3} + c}$

62. $\int \frac{2x+1}{\sqrt{x+4}}dx = \int \frac{2(u-4)+1}{\sqrt{u}}du$

$u = x+4 \longrightarrow u-4 = x$

$du = dx$

$= \int \frac{2u^1-7}{u^{1/2}}du$

$= \int (2u^{1/2} - 7u^{-1/2})du$

$= \frac{4}{3}u^{3/2} - 14u^{1/2} + C$

$= \boxed{\frac{4}{3}(x+4)^{3/2} - 14(x+4)^{1/2} + C}$

$\int c \cdot x^n dx$

$= c \cdot \frac{x^{n+1}}{n+1} + C_0$

$\int 7x^{-1/2}dx$

$= \frac{7x^{-1/2+1}}{-\frac{1}{2}+1}$

$= \frac{7x^{1/2}}{1/2} = 7x^{1/2} \cdot \frac{2}{1} = 14x^{1/2}$

Definite Integrals

66. $\int_{-2}^{4} x^2(x^3+8)^2 dx = \int_{x=-2\ (u=0)}^{x=4\ (u=72)} \frac{1}{3}u^2 du$

$u = x^3+8$

$du = 3x^2 dx$

$\frac{1}{3}du = x^2 dx$

$= \frac{1}{9}u^3 \Big|_{x=-2}^{x=4}$

$= \frac{1}{9}(x^3+8)^3 \Big|_{-2}^{4}$

$= \frac{1}{9}(4^3+8)^3 - \cancel{\frac{1}{9}((-2)^3+8)^3}$

$= \boxed{41,472}$

$$\int \frac{du}{u} = \ln\,|u| + K$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\,|f(x)| + K$$

$$\int \tan\,u\,du = -\ln\,|\cos\,u| + K$$

$$\int \cot\,u\,du = \ln\,|\sin\,u| + K$$

$$\int \sec\,u\,du = \ln\,|\sec\,u + \tan\,u| + K$$

$$\int \csc\,u\,du = \ln\,|\csc\,u - \cot\,u| + K$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$$

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx$$

$$= \ln|x| + C$$