

$$\int \frac{du}{u} = \ln |u| + K$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + K$$

$$\int \tan u du = -\ln |\cos u| + K$$

$$\int \cot u du = \ln |\sin u| + K$$

$$\int \sec u du = \ln |\sec u + \tan u| + K$$

$$\int \csc u du = \ln |\csc u - \cot u| + K$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$$

$$\begin{aligned} \int x^{-1} dx &= \int \frac{1}{x} dx \\ &= \ln |x| + c \end{aligned}$$

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx = \int -\frac{1}{u} du$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$-du = \sin x dx$$

$$= -\ln |u| + c$$

$$= \boxed{-\ln |\cos x| + c}$$

5.2

$$11. \int \frac{x^2 + 2x + 3}{x^3 + 3x^2 + 9x} dx = \int \frac{1}{3} \frac{du}{u}$$
$$= \frac{1}{3} \ln |x^3 + 3x^2 + 9x| + C$$

$$u = x^3 + 3x^2 + 9x$$

$$du = (3x^2 + 6x + 9) dx$$

$$du = 3(x^2 + 2x + 3) dx$$

$$\frac{1}{3} du = (x^2 + 2x + 3) dx$$

$$7. \int \frac{x}{x^2 + 1} dx = \int \frac{1}{2} \frac{du}{u}$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$= \frac{1}{2} \ln |x^2 + 1| + C$$

$$= \frac{1}{2} \ln(x^2 + 1) + C$$

$$9. \int \frac{x^2 - 4}{x} dx = \int \left(\frac{x^2}{x} - \frac{4}{x} \right) dx$$

~~$$u = x^2 - 4$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$~~

$$= \int \left(x - \frac{4}{x} \right) dx$$

$$= \boxed{\frac{1}{2}x^2 - 4 \ln|x| + C}$$

5.2

$$34. \int \frac{\csc^2 t}{\cot t} dt = \int -\frac{du}{u}$$

$$u = \cot t$$

$$du = -\csc^2 t dt$$

$$-du = \csc^2 t dt$$

$$= \boxed{-\ln|\cot t| + C}$$

$$\frac{5.4}{\int e^x dx = e^x + C} \quad [e^x]' = e^x$$

$$\frac{5.5}{\int a^x dx = \frac{1}{\ln a} \cdot a^x + C} \quad [a^x]' = a^x \cdot \ln a$$

$$\frac{5.4}{94. \int \frac{e^{1/x^2}}{x^3} dx = \int -\frac{1}{2} e^u du}$$

$$u = \frac{1}{x^2} = x^{-2}$$

$$du = -2x^{-3} dx$$

$$-\frac{1}{2} du = \frac{dx}{x^3}$$

$$= -\frac{1}{2} e^u + C$$

$$= \boxed{-\frac{1}{2} e^{1/x^2} + C}$$

$$104. \int \frac{e^{2x} + 2e^x + 1}{e^x} dx$$

$$= \int (e^x + 2 + e^{-x}) dx$$

$$= e^x + 2x + \int e^{-x} dx$$

$$= e^x + 2x + \int -e^u du$$

$$= \boxed{e^x + 2x - e^{-x} + C}$$

$$\left. \begin{aligned} e^{2x} &= e^{2x-x} \\ \frac{e^{2x}}{e^x} &= e^x \\ &= e^x \end{aligned} \right\}$$

$$\begin{aligned} u &= -x \\ du &= -dx \\ -du &= dx \end{aligned}$$

$$108. \int \ln(e^{2x-1}) dx = \int (2x-1) dx$$

$$\boxed{\begin{aligned} \log_a a^x &= x \\ a^{\log_a x} &= x \end{aligned}}$$

$$= \boxed{x^2 - x + C}$$

5.5

$$68. \int 2^{\sin x} \cos x dx = \int 2^u du$$

$$u = \sin x \\ du = \cos x dx$$

$$= \frac{1}{\ln 2} \cdot 2^u + C \\ = \frac{2^{\sin x}}{\ln 2} + C$$

$$64. \int_{-2}^0 (3^3 - 5^2) dx$$

$$= \int_{-2}^0 2 dx = 2x \Big|_{-2}^0 = 2(0) - 2(-2) \\ = 4$$

5.4

$$102. \int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx = \int \frac{2du}{u^2}$$

$$u = e^x + e^{-x}$$

$$du = (e^x - e^{-x})dx$$

$$2du = (2e^x - 2e^{-x})dx$$

$$= \int 2u^{-2} du$$

$$= -2u^{-1} + C$$

$$= \boxed{-\frac{2}{e^x + e^{-x}} + C}$$

5.9 Inverse Trig Functions

$$\frac{d}{dx} [\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx} [\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + c$$