5.9 Inverse Trig Functions

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$(\arctan x)' = \frac{1}{1+x^2}$

$(\arctan u)' = \frac{u'}{1+u^2}$

$\left(\arctan\frac{u}{a}\right)' = \frac{u'}{a\left(1+\left(\frac{u}{a}\right)^2\right)}$

$\frac{d}{dx}\left(\arctan\frac{u}{a}\right) = \frac{\frac{du}{dx}}{a+\frac{u^2}{a}}$

$\frac{1}{a}d\left(\arctan\frac{u}{a}\right) = \frac{1}{a}\frac{1}{a+\frac{u^2}{a}}du$

$\int d\left(\frac{1}{a}\arctan\frac{u}{a}\right) = \int\frac{du}{a^2+u^2}$

$\frac{1}{a}\arctan\frac{u}{a} + C = \int\frac{du}{a^2+u^2}$

2. $\int \frac{3dx}{\sqrt{1-4x^2}}$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$= \int \frac{3\,dx}{\sqrt{1^2-(2x)^2}} = \frac{3}{2}\int \frac{du}{\sqrt{1^2-u^2}}$

$u = 2x \quad a = 1$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$= \boxed{\frac{3}{2}\cdot \arcsin 2x + C}$

8. $\int_{\sqrt{3}}^{3} \frac{1}{9+x^2}\,dx$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$a = 3$

$u = x$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

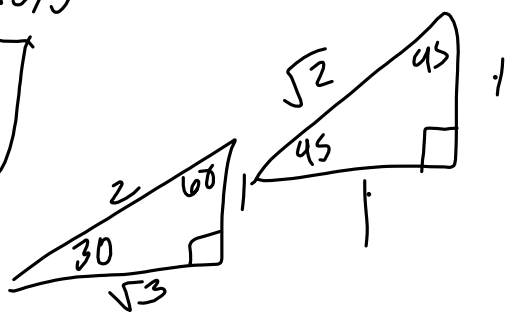
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$= \frac{1}{3}\arctan\frac{x}{3}\Big|_{\sqrt{3}}^{3}$

$= \frac{1}{3}\arctan(1) - \frac{1}{3}\arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{3}\left(\frac{\pi}{4}\right) - \frac{1}{3}\left(\frac{\pi}{6}\right)$

$1 = 3/3$ $\quad \frac{1}{\sqrt{3}} = \sqrt{3}/3$ $\quad 2 \cdot 2$ $\quad 2 \cdot 3$

$= \frac{3\pi}{36} - \frac{2\pi}{36} = \boxed{\frac{\pi}{36}}$



$$12.\ \int \frac{x^4-1}{x^2+1}\,dx$$

$$= \int \frac{(x^2-1)(x^2+1)}{x^2+1}\,dx$$

$$= \int (x^2-1)\,dx = \frac{1}{3}x^3 - x + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$$16.\ \int \frac{1}{x\sqrt{x^4-4}}\,dx$$

$$= \int \frac{1}{x\sqrt{(x^2)^2-2^2}}\,dx$$

$u = x^2$

$\frac{1}{2}du = 2x\,dx$

$u = x^4 - 4$

$du = 4x^3\,dx$

$$\int \frac{x\,dx}{x^2\sqrt{(x^2)^2-2^2}} = \frac{1}{2}\int \frac{du}{u\sqrt{u^2-2^2}}$$

$$= \frac{1}{2}\cdot\frac{1}{2}\operatorname{arcsec}\frac{x^2}{2} + C$$

$$= \frac{1}{4}\operatorname{arcsec}\frac{x^2}{2} + C$$

30. $\int \frac{x-2}{(x+1)^2+4}\,dx$

$v = x^2+2x+5$

$u = x+1$
$du = dx$

$dv = (2x+2)\,dv$
$= 2(x+1)\,dv$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$$= \int \frac{x+1-3}{(x+1)^2+4}\,dx = \int \frac{x+1}{x^2+2x+5}\,dx - \int \frac{3}{(x+1)^2+4}\,dx$$

$\frac{du}{u}$ $\qquad$ $\arctan u$

$u = x^2+2x+5$
$du = (2x+2)\,dx$
$\frac{1}{2}du = (x+1)\,dx$

$v = x+1$
$dv = dx$

$$= \frac{1}{2}\int \frac{du}{u} - 3\int \frac{dv}{v^2+2^2}$$

$$= \frac{1}{2}\ln\left|x^2+2x+5\right| - 3\cdot\frac{1}{2}\arctan\frac{x+1}{2} + C$$

$$= \frac{1}{2}\ln\left|x^2+2x+5\right| - \frac{3}{2}\arctan\frac{x+1}{2} + C$$