5.9 Inverse Trig Functions

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$
$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$$
$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$$\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$$

$$= \int \frac{dx}{(x-1)\sqrt{x^2-2x+1-1}}$$

$$= \int \frac{dx}{(x-1)\sqrt{(x-1)^2-1}}$$

$u = x-1$
$du = dx$

$$= \int \frac{du}{u\sqrt{u^2-1^2}} = \boxed{\operatorname{arcsec}|x-1| + c}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

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$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$$\int \frac{1}{3+25x^2}\,dx = \int \frac{1}{5}\cdot\frac{du}{3+u^2}$$

$u = 5x$ $\quad a=\sqrt{3}$

$du = 5dx$

$\frac{1}{5}du = dx$

$$= \boxed{\frac{1}{5}\cdot\frac{1}{\sqrt{3}}\arctan\frac{5x}{\sqrt{3}} + C}$$

$$\int \frac{4-x\,dx}{\sqrt{4-x^2}} = \underbrace{\int \frac{4\,dx}{\sqrt{4-x^2}}} + \underbrace{\int \frac{-x\,dx}{\sqrt{4-x^2}}}$$

$u = 4-x^2$

$du = -2x\,dx$

$\frac{1}{2}du = -x\,dx$

$$= 4\arcsin\frac{x}{2} + \int \frac{1}{2}\frac{du}{\sqrt{u}}$$

$$= 4\arcsin\frac{x}{2} + \frac{1}{2}\int u^{-1/2}\,du$$

$$= 4\arcsin\frac{x}{2} + \frac{1}{2}\cdot 2u^{1/2} + C$$

$$= \boxed{4\arcsin\frac{x}{2} + \sqrt{4-x^2} + C}$$

$$\int \frac{\arctan(x/2)}{4+x^2}\,dx = \int \frac{1}{2}u\,du = \frac{1}{4}u^2 + C$$

$$= \boxed{\frac{1}{4}\arctan^2\frac{x}{2} + C}$$

$\frac{1}{2}\int u\,du$

$\frac{1}{2}\cdot\frac{1}{2}u^2$

$$\frac{1}{2+3} \neq \frac{1}{2} + \frac{1}{3}$$

$u = \arctan\frac{x}{2}$

$du = \frac{1}{1+\left(\frac{x}{2}\right)^2}\cdot\frac{1}{2}\,dx$

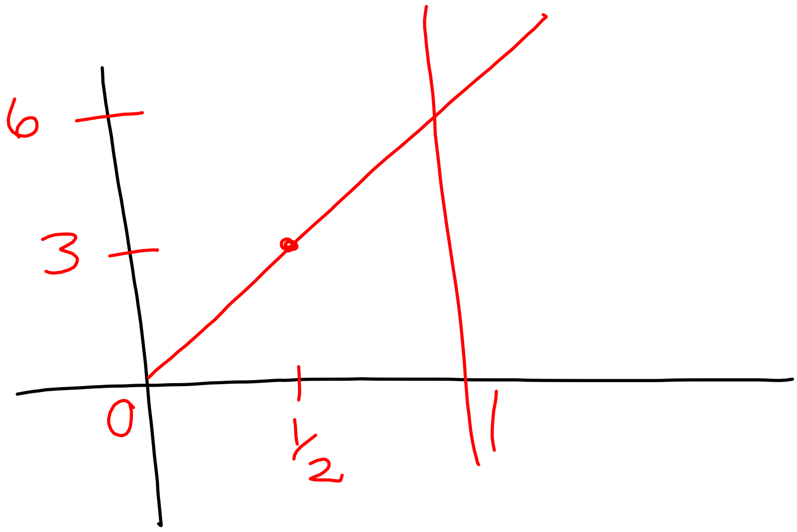
$du = \frac{dx}{\left(1+\frac{x^2}{4}\right)2}$

$du = \frac{dx}{2+\frac{x^2}{2}}\cdot\frac{2}{2} = \frac{2\,dx}{4+x^2}$

$\frac{1}{2}du = \frac{dx}{4+x^2}$

$$\int_1^2 x\sqrt{x-1}\,dx \qquad \int \text{blah}\,dx$$

$$\int_{\square}^{\square} \square\, d\square \qquad \int \square\, d\square$$



$$F(x) = \int_a^{g(x)} f(t)\,dt$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$F'(x) = f(g(x)) \cdot g'(x)$$

$$\int_{g(x)}^{h(x)} = \int_{g(x)}^{a} + \int_a^{h(x)} = -\int_a^{g(x)} + \int_a^{h(x)}$$

MVT $y = x^2$, $[0,1]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$x^2 = \int_0^1 x^2\,dx$$

$$x^2 = \frac{1}{3}(1)^3 - \frac{1}{3}(0)^3$$

$$x = \sqrt{1/3}$$

$$\int_0^1 e^x \sin(e^x)\,dx = \int_{e^0\;x=0}^{e^1\;x=1} \sin u\,du$$

$u = e^x$

$du = e^x\,dx$

$$= \int_1^e \sin u\,du$$

$$= -\cos e - (-\cos 1)$$