8.1 Basic Integration Rules

$$\int \frac{4}{x^2+9}\,dx = \boxed{\frac{4}{3}\arctan\frac{x}{3}+C}$$

$$\int \frac{4x}{x^2+9}\,dx$$

$u = x^2+9$
$du = 2x\,dx$
$2du = 4x\,dx$

$$= \int \frac{2du}{u} = \boxed{2\ln(x^2+9)+C}$$

$$\int \frac{4x^2}{x^2+9}\,dx = 4\left[\int \frac{x^2+9-9}{x^2+9}\,dx\right]$$

$$= 4\int \frac{x^2+9}{x^2+9}\,dx - 4\int \frac{9}{x^2+9}\,dx$$

$$= 4\int dx - 36\int \frac{dx}{x^2+9}$$

$$= \boxed{4x - 12\arctan\frac{x}{3}+C}$$

$$\int \frac{1+e^x-e^x}{1+e^x}\,dx$$

$$= \int dx - \int \frac{e^x\,dx}{1+e^x}$$

$u = 1+e^x$
$du = e^x\,dx$

$$\int \frac{du}{u}$$

$$= \boxed{x - \ln(1+e^x)+C}$$

$$\int \tan^2 2x \, dx$$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$$= \int (\sec^2 2x - 1) \, dx$$

$$= \boxed{\frac{1}{2}\tan 2x - x + C}$$

$$\int \cot x \ln(\sin x) \, dx = \int u \, du = \boxed{\frac{1}{2}\left[\ln(\sin x)\right]^2 + C}$$

$u = \ln(\sin x)$

$du = \frac{1}{\sin x} \cdot \cos x \, dx = \cot x \, dx$

8.2 Integration by Parts

$$\frac{d}{dx}[uv] = u \cdot \frac{d}{dx}[v] + v \cdot \frac{d}{dx}[u]$$

$$= uv' + vu'$$

Integrating both sides w.r.t. x yields:

$$uv = \int uv'dx + \int vu'dx$$

$$uv = \int udv + \int vdu$$

Rearranging yields:

$$\boxed{\int udv = uv - \int vdu}$$

$$\int udv = uv - \int vdu$$

$$\int xe^x dx$$

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$$\int xe^x dx = xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$$

6. $\int x^2 e^{2x}\,dx = \frac{1}{2}x^2 e^{2x} - \underbrace{\int x e^{2x}\,dx}$

$u = x^2 \qquad dv = e^{2x}\,dx$

$du = 2x\,dx \qquad v = \frac{1}{2}e^{2x}$

$u = x \qquad dv = e^{2x}\,dx$

$du = dx \qquad v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}x^2 e^{2x} - \left[\frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x}\,dx\right]$$

$$= \boxed{\frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + C}$$

16. $\int x^4 \ln x\,dx = \frac{1}{5}x^5 \ln x - \int \frac{1}{5}x^4\,dx$

$u = \ln x \qquad dv = x^4\,dx$

$du = \frac{dx}{x} \qquad v = \frac{1}{5}x^5$

$$= \boxed{\frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C}$$

34. $\int 4\arccos x\,dx = 4x\arccos x + \int \frac{+4x}{\sqrt{1-x^2}}dx$

$u = 1-x^2$

$du = -2x\,dx$

$-2du = 4x\,dx$

$u = \arccos x \qquad dv = 4dx$

$du = \frac{-1}{\sqrt{1-x^2}}dx \qquad v = 4x$

$= 4x\arccos x - \int \frac{2du}{\sqrt{u}}$

$\int 2u^{-1/2}du$

$= 4x\arccos x - 4u^{1/2} + C$

$= \boxed{4x\arccos x - 4\sqrt{1-x^2} + C}$