Solve the differential equation.

40. $\dfrac{dy}{dx} = x^2\sqrt{x-1}$

$\int dy = \int x^2\sqrt{x-1}\,dx$ $\quad u = x-1 \quad x = u+1$, $du = dx$

$y = \int (u+1)^2 u^{1/2}\,du$ $\quad \int x^2 (x-1)^{1/2}\,dx$??

$= \int (u^2+2u+1)u^{1/2}\,du$

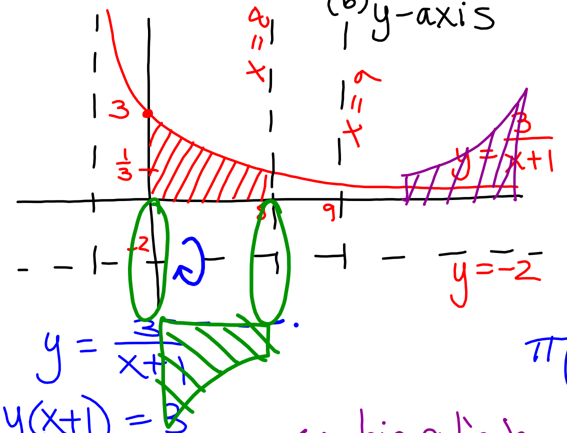
$= \int \left(u^{5/2} + 2u^{3/2} + u^{1/2}\right)du$

$= \frac{2}{7}u^{7/2} + \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

$$\boxed{y = \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C}$$

26. $y = \dfrac{3}{x+1}$, $y=0$, $x=0$, $x=8$

set up the integrals to revolve about (a) x-axis (b) y-axis (c) $x=9$ (d) $y=-2$



(a) $\int_0^8 \pi \left(\frac{3}{x+1}\right)^2 dx$

(b) $\cancel{\int_0^3 \pi \left(\frac{3}{\cdot}\right)^2 dy}$

$\pi(8)^2 \cdot \frac{1}{3} + \int_{1/3}^{3} \pi\left(\frac{3}{y}-1\right)^2 dy$

$y = \frac{3}{x+1}$

$y(x+1) = 3$

$x+1 = \frac{3}{y}$

$x = \frac{3}{y} - 1$

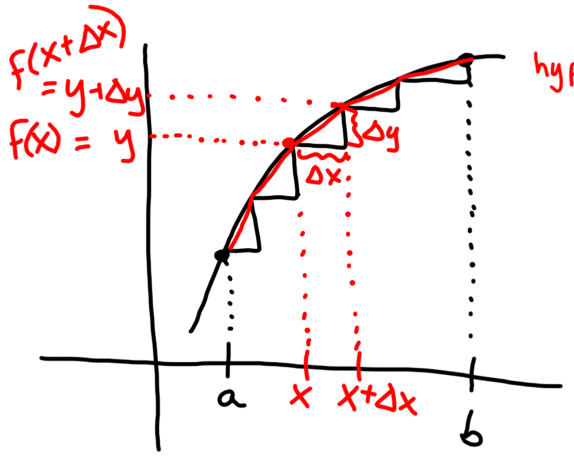
(c) big cylinder − (bowl + little bottom cylinder)

$\pi(9)^2\, 3 - \int_{1/3}^{3} \pi\left(9 - \left(\frac{3}{y}-1\right)\right)^2 dy - \pi(1)^2 \cdot \frac{1}{3}$

(d) $\int_0^8 \pi \left(\frac{3}{x+1} - (-2)\right)^2 dx - \pi(2)^2 \cdot 8$

6.4 - Arc Length & Surfaces of Revolution

The arc length s of a smooth curve f from a to b is $s = \int_a^b \sqrt{1+[f'(x)]^2}\, dx$



hypotenuse has length $\sqrt{(\Delta x)^2+(\Delta y)^2}$

$= \sqrt{(\Delta x)^2\left[1+\left(\frac{\Delta y}{\Delta x}\right)^2\right]}$

$= \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2} \cdot \Delta x$

MVT $f(x)-f(x+\Delta x) = f'(c)\cdot \Delta x$

$\frac{\Delta y}{\Delta x} = f'(c)$

6. $y = \frac{3}{2}x^{2/3}+4, \quad [1,27]$

$s = \int_a^b \sqrt{1+(f'(x))^2}\cdot dx$ $\qquad f'(x) = y' = x^{-1/3}$

$= \int_1^{27} \sqrt{1+\left(\frac{1}{\sqrt[3]{x}}\right)^2}\, dx = \int_1^{27} \sqrt{\frac{1}{(\sqrt[3]{x})^2}\left(\sqrt[3]{x^2}+1\right)}\, dx$

$= \int_1^{27} \frac{1}{\sqrt[3]{x}} \sqrt{\sqrt[3]{x^2}+1}\, dx = \int_1^{27} x^{-1/3}\sqrt{x^{2/3}+1}\, dx$

$u = x^{2/3}+1$

$du = \frac{2}{3}x^{-1/3}\, dx$

$\frac{3}{2}du = x^{-1/3}\, dx$

$= \frac{3}{2}\int_{x=1}^{x=27} u^{1/2}\, du$

$= \frac{3}{2}\cdot\frac{2}{3}\left(x^{2/3}+1\right)^{3/2}\Big|_{x=1}^{27}$