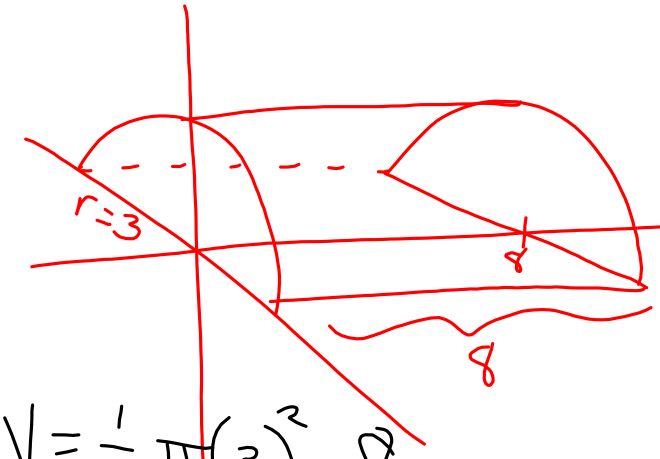
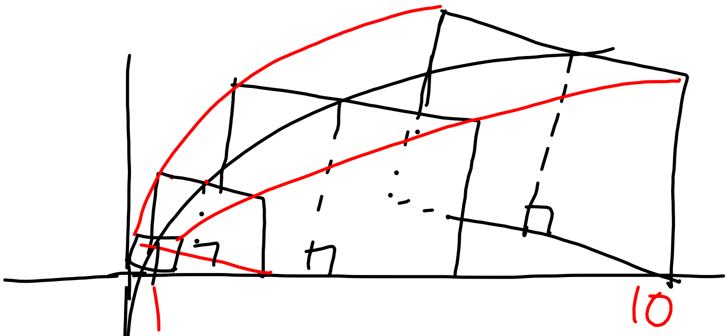
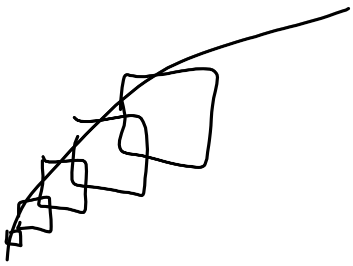
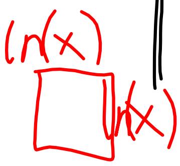


$$V = \frac{1}{2}\pi(3)^2 \cdot 8$$





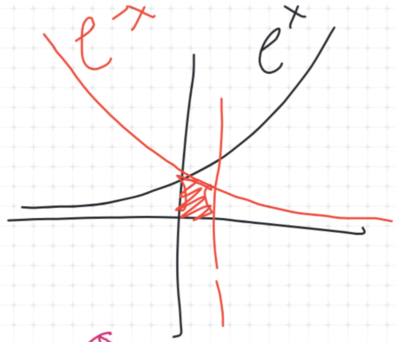
solid whose cross sections are squares of edge length $\ln x$

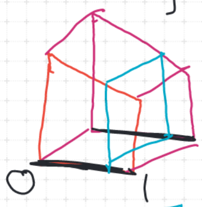


$$V = \int_1^{10} [\ln x]^2 \, dx$$

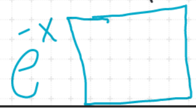
The base of a solid S is the region bounded by the curve $y = e^{-x}$, the x-axis, the y-axis, and the line $x = 1$. Cross-sections perpendicular to the x-axis are squares.

Determine the *exact* volume of solid S.





parallel w/ x-axis

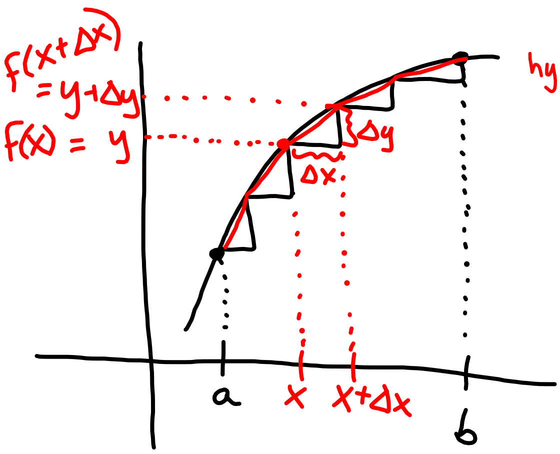


$$V = \int_0^1 \left(e^{-x}\right)^2 dx$$

6.4 - Arc Length & Surfaces of Revolution

The arc length s of a smooth curve f from a to b is

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$



hypotenuse has length $\sqrt{(\Delta x)^2 + (\Delta y)^2}$

$$= \sqrt{(\Delta x)^2\left[1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right]}$$

$$= \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} \cdot \Delta x$$

MVT $f(x) - f(x+\Delta x) = f'(c) \cdot \Delta x$

$$\frac{\Delta y}{\Delta x} = f'(c)$$

6. $y = \frac{3}{2}x^{2/3} + 4, \ [1, 27]$

$S = \int_a^b \sqrt{1+(f'(x))^2} \cdot dx \qquad f'(x) = y' = x^{-1/3}$

$= \int_1^{27} \sqrt{1+\left(\frac{1}{\sqrt[3]{x}}\right)^2}\, dx = \int_1^{27} \sqrt{\frac{1}{(\sqrt[3]{x})^2}\left(\sqrt[3]{x^2}+1\right)}\, dx$

$= \int_1^{27} \frac{1}{\sqrt[3]{x}} \sqrt{\sqrt[3]{x^2}+1}\, dx = \int_1^{27} x^{-1/3}\sqrt{x^{2/3}+1}\, dx$

$u = x^{2/3}+1$

$du = \frac{2}{3}x^{-1/3}dx$

$\frac{3}{2}du = x^{-1/3}dx$

$= \frac{3}{2}\int_{x=1}^{x=27} u^{1/2}\, du$

$= \frac{3}{2} \cdot \frac{2}{3}\left(x^{2/3}+1\right)^{3/2} \Big|_{x=1}^{27}$

$\sqrt{\left(\sqrt[3]{27}^2+1\right)^3} - \sqrt{\sqrt[3]{1}^2+1}^3$

$\boxed{10\sqrt{10} - 2\sqrt{2}}$

18. $y = \ln x \quad , [1, 5]$

$a = 1 \quad f(x) = \ln x$

$b = 5 \quad f'(x) = \frac{1}{x}$

$S = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\, dx$

$S = \int_1^5 \sqrt{1+\left(\frac{1}{x}\right)^2}\, dx = \int_1^5 \sqrt{\frac{1}{x^2}\left(x^2+1\right)}\, dx$

$= \int_1^5 \frac{\sqrt{x^2+1}}{x}\, dx$

$x = \tan\theta \quad dx = \sec^2\theta\, d\theta$

$x^2+1 = \tan^2\theta + 1$

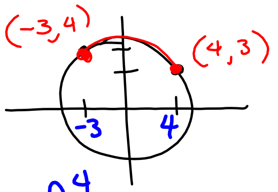
$= \int_{x=1}^{x=5} \frac{\sec\theta}{\tan\theta} \cdot \sec^2\theta\, d\theta$

$\sqrt{x^2+} = \sqrt{\tan^2\theta+1}$

$= \sqrt{\sec^2\theta} = \sec\theta$

$= \int \frac{\cos^4\theta}{\sin\theta}\, d\theta = \ldots$

32. Find arc length from $(-3,4)$ clockwise to $(4,3)$ along the circle $x^2+y^2=25$.



$$f(x) = y = \sqrt{25-x^2}$$

$$f'(x) = \frac{1}{2}(25-x^2)^{-1/2}\cdot(-2x)$$

$$f'(x) = \frac{-x}{\sqrt{25-x^2}}$$

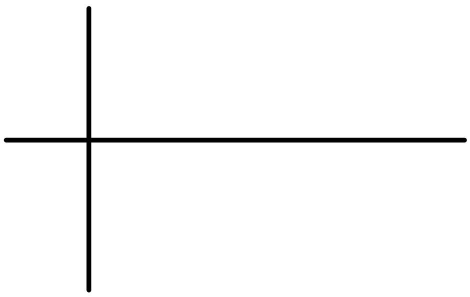
$$s = \int_{-3}^{4} \sqrt{1+\left[\frac{-x}{\sqrt{25-x^2}}\right]^2}\,dx$$

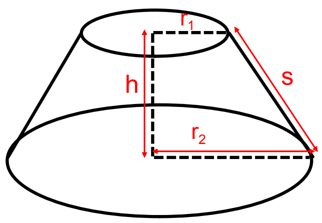
$$= \int_{-3}^{4} \sqrt{1+\frac{x^2}{25-x^2}}\,dx = \int_{-3}^{4} \sqrt{\frac{25-x^2}{25-x^2}+\frac{x^2}{25-x^2}}\,dx$$

$$= \int_{-3}^{4} \sqrt{\frac{25}{25-x^2}}\,dx = \int_{-3}^{4} \frac{5}{\sqrt{25-x^2}}\,dx$$

$$= 5\arcsin\frac{x}{5}\Big|_{-3}^{4} = \boxed{5\arcsin\frac{4}{5} - 5\arcsin\left(\frac{-3}{5}\right)}$$

Area of a Surface of Revolution



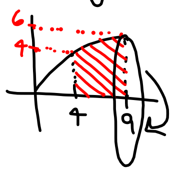


Truncated Cone:
$A = 2\pi \cdot r_{avg} \cdot s$

$$s = 2\pi \int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$S = 2\pi \int_a^b r(x)\sqrt{1+[f'(x)]^2}\, dx$$

34. $y = 2\sqrt{x}$, $[4,9]$ $\quad r(x) = 2\sqrt{x}$; $f'(x) = \frac{1}{\sqrt{x}}$
revolve about x-axis



$$\int_4^9 2\pi(2\sqrt{x})\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\, dx$$

$$= \int_4^9 4\pi\sqrt{x}\sqrt{1+\frac{1}{x}}\, dx = \int_4^9 4\pi\sqrt{x}\sqrt{\frac{x+1}{x}}\, dx$$

$$= \int_4^9 4\pi\cancel{\sqrt{x}}\cdot\frac{\sqrt{x+1}}{\cancel{\sqrt{x}}}\, dx = \int_4^9 4\pi\sqrt{x+1}\, dx \qquad \begin{aligned} u &= x+1 \\ du &= dx \end{aligned}$$

$$= \int_5^{10} 4\pi u^{1/2}\, du = 4\pi\cdot\frac{2}{3}u^{3/2}\Big|_5^{10} = \frac{8\pi}{3}(10)^{3/2} - \frac{8\pi}{3}(5)^{3/2}$$

$$= \frac{80\pi}{3}\sqrt{10} - \frac{40\pi}{3}\sqrt{5}$$

7.3 Trigonometric Integrals

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 2\cos^2 x - 1$

$\qquad\ = 1 - 2\sin^2 x$

4. $\int \cos^3 x \sin^4 x\, dx$