7.3 Trigonometric Integrals

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

4. $\int \cos^3 x \sin^4 x \, dx = \int \underbrace{\cos^2 x}\sin^4 x \cos x \, dx$

$= \int (1-\sin^2 x)\sin^4 x \cos x \, dx$

$= \int (\sin^4 x - \sin^6 x)\cos x \, dx = \int (u^4 - u^6)\, du$

$u = \sin x$

$du = \cos x \, dx$

$= \frac{1}{5}u^5 - \frac{1}{7}u^7 + C$

$= \boxed{\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$

12. $\int \sin^2 2x \, dx$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\sin^2 2x = \frac{1-\cos 2(2x)}{2}$

$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 4x\right) dx$

$= \boxed{\frac{1}{2}x - \frac{1}{8}\sin 4x + C}$

$u = 4x$

$du = 4\,dx$

$\frac{1}{4}du = dx$

$\int \cos 4x \, dx$

$= \frac{1}{4}\int \cos u \, du$

$= \frac{1}{4}\sin u + C$

26. $\int \tan^2 x \, dx$

$= \int (\sec^2 x - 1) \, dx$

$= \boxed{\tan x - x + C}$

38. $\int \frac{\tan^2 x}{\sec^5 x} dx = \int \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^5 x}} dx$

$= \int \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^5 x}{1} dx = \int \sin^2 x \cos^3 x \, dx$

$= \int \sin^2 x \underbrace{\cos^2 x} \cos x \, dx$

$= \int \sin^2 x (1 - \sin^2 x) \cos x \, dx$

$= \int (\sin^2 x - \sin^4 x) \cos x \, dx = \int (u^2 - u^4) du$

$u = \sin x$
$du = \cos x \, dx$

$= \boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$

16. $\int x^2 \sin^2 x\,dx = \int x^2\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$= \int \frac{1}{2}x^2\,dx - \int \frac{1}{2}x^2 \cos 2x\,dx$ $\quad u = \frac{1}{2}x^2 \quad dv = \cos 2x\,dx$; $du = x\,dx \quad v = \frac{1}{2}\sin 2x$

$= \frac{1}{6}x^3 - \left(\frac{1}{4}x^2 \sin 2x - \int \frac{1}{2}x \sin 2x\,dx\right)$ $\quad u = \frac{1}{2}x \quad dv = \sin 2x\,dx$; $du = \frac{1}{2}dx \quad v = -\frac{1}{2}\cos 2x$

$= \frac{1}{6}x^3 - \frac{1}{4}x^2 \sin 2x + \left(-\frac{1}{4}x\cos 2x - \int -\frac{1}{4}\cos 2x\,dx\right)$

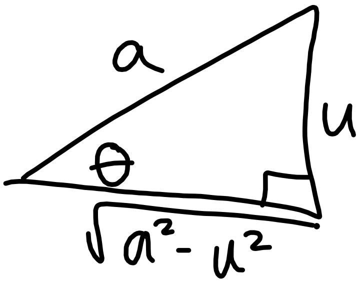
$= \frac{1}{6}x^3 - \frac{1}{4}x^2 \sin 2x - \frac{1}{4}x\cos 2x + \int \frac{1}{4}\cos 2x\,dx$

$= \boxed{\frac{1}{6}x^3 - \frac{1}{4}x^2 \sin 2x - \frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C}$

7.4 Trig Substitution

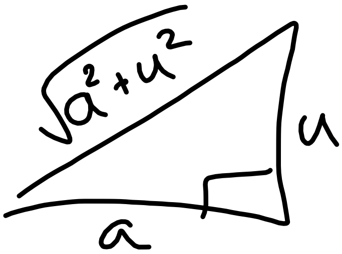
$\sqrt{a^2 - u^2} = a\cos\theta$ $\quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$u = a\sin\theta$



$\sqrt{a^2 + u^2} = a\sec\theta$, $\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

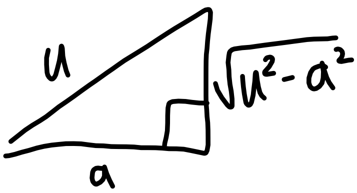
$u = a\tan\theta$



$\sqrt{u^2 - a^2} = \begin{cases} a\tan\theta, & u > a \\ -a\tan\theta, & u < -a \end{cases}$

$u = a\sec\theta$

$0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$



6. $\displaystyle\int \frac{10}{x^2\sqrt{25-x^2}}\,dx = \int \frac{10\cdot 5\cos\theta\, d\theta}{(5\sin\theta)^2\sqrt{25-(5\sin\theta)^2}}$

$x = 5\sin\theta$

$dx = 5\cos\theta\, d\theta$

$\displaystyle = \int \frac{\overset{2}{\cancel{50}}\cos\theta\, d\theta}{\cancel{25}\sin^2\theta\sqrt{25-25\sin^2\theta}}$

$25(1-\sin^2\theta)$

$\displaystyle = \int \frac{2\cos\theta\, d\theta}{\sin^2\theta\sqrt{25\cos^2\theta}} = \int \frac{2\cancel{\cos\theta}\, d\theta}{\sin^2\theta \cdot 5\cancel{\cos\theta}}$

$\displaystyle = \int \frac{2}{5}\csc^2\theta\, d\theta = -\frac{2}{5}\cot\theta + c$

$\displaystyle = \boxed{-\frac{2}{5}\cdot\frac{\sqrt{25-x^2}}{x} + c}$

$x = 5\sin\theta$

$\frac{x}{5} = \sin\theta$