

$$12. \int \frac{x^3 dx}{\sqrt{x^2-4}} = \int \frac{(2\sec\theta)^3 \cdot 2\sec\theta\tan\theta d\theta}{\sqrt{4\sec^2\theta-4}}$$

$x = 2\sec\theta$
 $x^2 = 4\sec^2\theta$
 $dx = 2\sec\theta\tan\theta d\theta$

$\left\{ \begin{array}{l} \sqrt{4(\sec^2\theta-1)} \\ 2\sqrt{\sec^2\theta-1} \end{array} \right.$

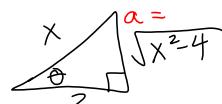
$$\int \frac{16\sec^4\theta\tan\theta d\theta}{2\tan\theta} = \int 8\sec^4\theta d\theta =$$

$$\int 8\sec^2\theta \sec^2\theta d\theta = \int 8(\tan^2\theta+1) \sec^2\theta d\theta$$

$$\begin{aligned} u &= \tan\theta \\ du &= \sec^2\theta d\theta \end{aligned} \quad = \int (8u^2 + 8)du = \frac{8}{3}u^3 + 8u + C =$$

$$= \frac{8}{3}\tan^3\theta + 8\tan\theta + C$$

$$\begin{aligned} x &= 2\sec\theta \\ \frac{x}{2} &= \sec\theta \end{aligned} \quad = \frac{8}{3}\left(\frac{\sqrt{x^2-4}}{2}\right)^3 + 8 \cdot \frac{\sqrt{x^2-4}}{2} + C$$



$$\begin{aligned} a^2 + 2^2 &= x^2 \\ a^2 &= x^2 - 4 \end{aligned}$$

$$= \boxed{\frac{1}{3}(x^2-4)^{3/2} + 4(x^2-4)^{1/2} + C}$$

$$16. \int \frac{x^2 dx}{(1+x^2)^2} = \int \frac{\tan^2\theta \cdot \sec^2\theta d\theta}{(1+\tan^2\theta)^2} = \int \frac{\tan^2\theta \sec^2\theta d\theta}{(\sec^2\theta)^2}$$

$$\begin{aligned} x &= \tan\theta \\ dx &= \sec^2\theta d\theta \end{aligned} \quad = \int \frac{\tan^2\theta d\theta}{\sec^2\theta} = \int \frac{\sin^2\theta}{\cos^2\theta} d\theta =$$

$$= \int \frac{\sin^2\theta \cdot \cos^3\theta}{\cos^3\theta} d\theta =$$

$$= \int \sin^2\theta d\theta = \int (\frac{1}{2} - \frac{1}{2}\cos 2\theta) d\theta =$$

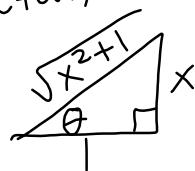
$$= \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C \quad x = \tan\theta$$

$$= \frac{1}{2}\arctan x - \frac{1}{4} \cdot 2\sin\theta\cos\theta + C \quad \tan^{-1}(x) = \tan^{-1}(\tan\theta)$$

$$= \frac{1}{2}\arctan x - \frac{1}{2} \cdot \frac{x}{\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}} + C$$

$$= \boxed{\frac{1}{2}\arctan x - \frac{x}{2(x^2+1)} + C}$$

$$\begin{aligned} x &= \tan\theta \\ \tan^{-1}(x) &= \tan^{-1}(\tan\theta) \\ \arctan x &= \theta \end{aligned}$$



$$\begin{aligned}
 30. \int \frac{dx}{x\sqrt{4x^2+16}} &= \int \frac{x \sec^2 \theta d\theta}{x \tan \theta \sqrt{4(2\tan^2 \theta)^2 + 16}} \\
 x = 2\tan \theta & \qquad \qquad \qquad = \int \frac{\sec^2 \theta d\theta}{\tan \theta \sqrt{16\tan^2 \theta + 16}} \\
 dx = 2\sec^2 \theta d\theta & \qquad \qquad \qquad = \int \frac{\sec^2 \theta d\theta}{\tan \theta \sqrt{16(\tan^2 \theta + 1)}} = \\
 x^2 = 4\tan^2 \theta & \qquad \qquad \qquad = \int \frac{\sec^2 \theta d\theta}{\tan \theta (4\sec \theta)} = \\
 = \int \frac{\sec^2 \theta d\theta}{\tan \theta \sqrt{16\sec^2 \theta}} & \qquad \qquad \qquad = \int \frac{1}{4\sin \theta} d\theta = \int \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{4\sin \theta} d\theta \\
 = \int \frac{\sec \theta d\theta}{4\tan \theta} & \qquad \qquad \qquad = \int \frac{\sin \theta d\theta}{4(1-\cos^2 \theta)} \\
 = \int \frac{d\theta}{4\sin \theta} \cdot \frac{\sin \theta}{\sin \theta} & \qquad \qquad \qquad = \int \frac{-du}{4(1-u^2)} = \int \frac{1}{4(u^2-1)} du \\
 u = \cos \theta & \qquad \qquad \qquad = \int \frac{1}{a+b} - \int \frac{1}{a} + \int \frac{1}{b}
 \end{aligned}$$