12. $\displaystyle\int \frac{x^3\,dx}{\sqrt{x^2-4}} = \int \frac{(2\sec\theta)^3 \cdot 2\sec\theta\tan\theta\,d\theta}{\sqrt{4\sec^2\theta-4}}$

$\left\{ \sqrt{4(\sec^2\theta-1)} \quad 2\sqrt{\sec^2\theta-1} \quad 2\sqrt{\tan^2\theta} \right\}$

$x = 2\sec\theta$

$x^2 = 4\sec^2\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\displaystyle\int \frac{\cancel{16}\,8\sec^4\theta\tan\theta\,d\theta}{\cancel{2}\,\cancel{\tan\theta}} = \int 8\sec^4\theta\,d\theta =$

$\displaystyle = \int 8\sec^2\theta\sec^2\theta\,d\theta = \int 8(\tan^2\theta+1)\sec^2\theta\,d\theta$

$u = \tan\theta$

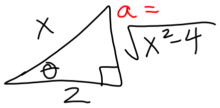
$du = \sec^2\theta\,d\theta$

$\displaystyle = \int (8u^2+8)\,du = \frac{8}{3}u^3 + 8u + C =$

$= \frac{8}{3}\tan^3\theta + 8\tan\theta + C$

$x = 2\sec\theta$

$\frac{x}{2} = \sec\theta$



$a^2 + 2^2 = x^2$

$a^2 = x^2 - 4$

$= \frac{8}{3}\left(\frac{\sqrt{x^2-4}}{2}\right)^3 + 8\cdot\frac{\sqrt{x^2-4}}{2} + C$

$= \boxed{\frac{1}{3}(x^2-4)^{3/2} + 4(x^2-4)^{1/2} + C}$

16. $\displaystyle\int \frac{x^2\,dx}{(1+x^2)^2} = \int \frac{\tan^2\theta\cdot\sec^2\theta\,d\theta}{(1+\tan^2\theta)^2} = \int \frac{\tan^2\theta\sec^2\theta\,d\theta}{(\sec^2\theta)^2}$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\displaystyle = \int \frac{\tan^2\theta\,d\theta}{\sec^2\theta} = \int \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}\,d\theta =$

$\displaystyle = \int \frac{\sin^2\theta}{\cancel{\cos^2\theta}}\cdot\frac{\cancel{\cos^2\theta}}{1}\,d\theta =$

$\displaystyle = \int \sin^2\theta\,d\theta = \int\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta =$

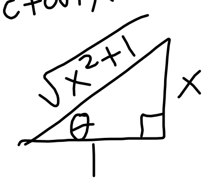
$= \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$

$x = \tan\theta$

$\tan^{-1}(x) = \tan^{-1}(\tan\theta)$

$\arctan x = \theta$

$= \frac{1}{2}\arctan x - \frac{1}{4}\cdot 2\underbrace{\sin\theta}\underbrace{\cos\theta} + C$



$= \frac{1}{2}\arctan x - \frac{1}{2}\cdot\frac{x}{\sqrt{x^2+1}}\cdot\frac{1}{\sqrt{x^2+1}} + C$

$= \boxed{\frac{1}{2}\arctan x - \frac{x}{2(x^2+1)} + C}$

30. $\displaystyle\int \frac{dx}{x\sqrt{4x^2+16}} = \int \frac{\cancel{2}\sec^2\theta\, d\theta}{\cancel{2}\tan\theta\sqrt{4(2\tan\theta)^2+16}}$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$x^2 = 4\tan^2\theta$

$$= \int \frac{\sec^2\theta\, d\theta}{\tan\theta\sqrt{16\tan^2\theta+16}} = \int \frac{\sec^2\theta\, d\theta}{\tan\theta\sqrt{16(\tan^2\theta+1)}} =$$

$$= \int \frac{\sec^2\theta\, d\theta}{\tan\theta\sqrt{16\sec^2\theta}} = \int \frac{\sec^2\theta\, d\theta}{\tan\theta(4\sec\theta)} =$$

$$= \int \frac{\sec\theta\, d\theta}{4\tan\theta} = \int \frac{\frac{1}{\cos\theta}\, d\theta}{\frac{4\sin\theta}{\cos\theta}} = \int \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{4\sin\theta}\, d\theta$$

$$= \int \frac{d\theta}{4\sin\theta} \cdot \frac{\sin\theta}{\sin\theta} = \int \frac{\sin\theta\, d\theta}{4\sin^2\theta} = \int \frac{\sin\theta\, d\theta}{4(1-\cos^2\theta)}$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$= \int \frac{-du}{4(1-u^2)} = \int \frac{1}{4(u^2-1)}\, du$$

$$\frac{1}{a+b} \cancel{=} \frac{1}{a} + \frac{1}{b}$$