Integral Calculus - Review

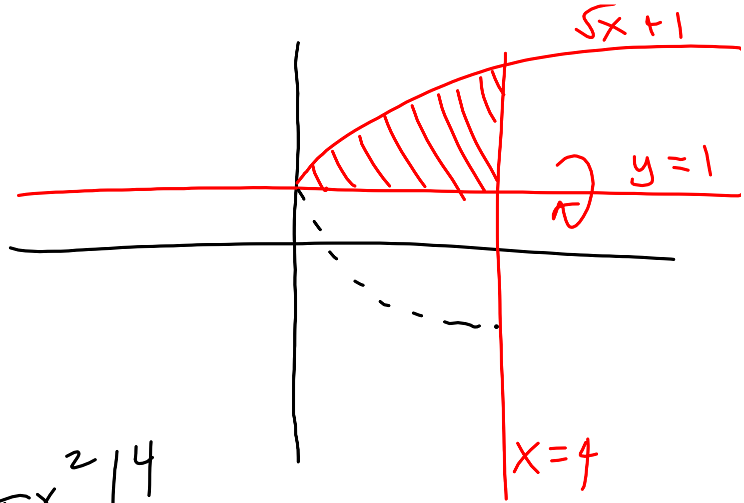
January 29, 2018

A region is enclosed by the line $y = 1$, the line $x = 4$ and the curve $y = \sqrt{x} + 1$.

What is the volume of the solid generated when this region is rotated around the line $y = 1$?

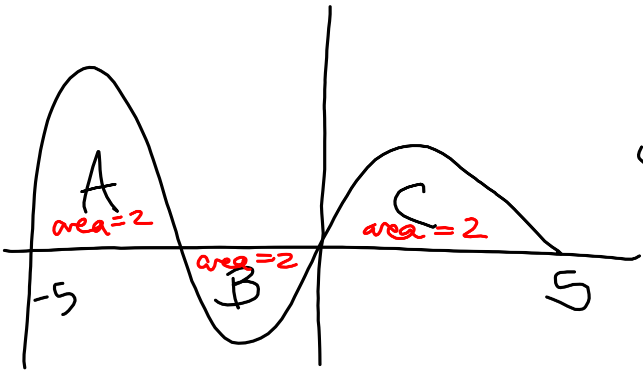
Express your answer in terms of π.

[ ] cubic units



$$\int_0^4 \pi(\sqrt{x})^2 dx$$

$$= \int_0^4 \pi x \, dx = \frac{\pi x^2}{2}\Big|_0^4$$



$$\int_{-5}^{5} (f(x)+1)\,dx$$

$$= \int_{-5}^{5} f(x)\,dx + \int_{-5}^{5} 1\,dx$$

$$2 - 2 + 2 + x\Big|_{-5}^{5}$$

$$2 + 5 + (+5)$$

$$= \boxed{12}$$

$$\int \sin^2 x \cos^2 x \, dx$$

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos 2x = 2\cos^2 x - 1$

$\frac{\cos 2x + 1}{2} = \cos^2 x$

$$\int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$$

$$= \frac{1}{4}\int \left(1 - \cos^2 2x\right) dx$$

$$= \int \frac{1}{4} dx - \frac{1}{4}\int \cos^2 2x \, dx$$

$$= \int \frac{1}{4} dx - \frac{1}{4}\int \left(\frac{\cos 4x + 1}{2}\right) dx$$

$\frac{\cos 4x}{2} + \frac{1}{2}$

$$= \int \frac{1}{4} dx - \frac{1}{4}\int \frac{1}{2} dx - \frac{1}{8}\int \cos 4x \, dx$$

$$\frac{1}{4}x - \frac{1}{8}x - \frac{1}{32}\sin 4x + C$$

$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \cdot \frac{dx}{x}$$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{dx}{x} \qquad v = \frac{1}{2}x^2$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$$\int \tan^3 x \sec^3 x \, dx$$

$$\int \underbrace{\tan^2 x}_{\sec^2 x - 1} \sec^2 x \cdot \sec x \tan x \, dx$$

$u = \sec x$

$$\int (u^4 - u^2) \, du$$

7.5 Partial Fractions

$$\int \frac{1}{x^2 - 5x + 6} dx = \int \frac{-1}{x-3} dx + \int \frac{1}{x-2} dx$$

$$= -\ln|x-3| + \ln|x-2| + c$$

$$\frac{1}{\underset{(x-3)(x-2)}{x^2 - 5x + 6}} = \frac{A}{x-3} + \frac{B}{x-2}.$$

$$= \frac{\overset{Ax - 2A + Bx - 3B}{A(x-2) + B(x-3)}}{(x-3)(x+2)} = \frac{(A+B)x + (-2A - 3B)}{(x-3)(x+2)}$$

$A + B = 0, \Rightarrow A = -B$

$-2A - 3B = 1 \Rightarrow -2(-B) - 3B = 1$

$-B = -1$

$B = 1 \quad A = -1$

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$$

$$x\left(x^2+2x+1\right)$$

$$x(x+1)(x+1)$$

$$\frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\int \frac{2x^3-4x-8}{(x^2-x)(x^2+4)}\,dx$$

$$\frac{2x^3-4x-8}{x(x-1)(x^2+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+4}$$

$$= \frac{A}{x}\cdot\frac{\overset{Ax-A}{(x-1)(x^2+4)}}{(x-1)(x^2+4)} + \frac{B}{x-1}\cdot\frac{x}{x}\,\frac{(x^2+4)}{x^2+4} + \frac{(Cx+D)}{x^2+4}\cdot\frac{(x^2+x)}{x(x+1)}$$

$$= \frac{Ax^3+4Ax-Ax^2-4A+Bx^3+4Bx+Cx^3+Cx^2+Dx^2+Dx}{x(x-1)(x^2+4)}$$

$$= \frac{(A+B+C)x^3+(-A+C+D)x^2+(4A+4B+D)x+(-4A)}{x(x-1)(x^2+4)}$$

$A+B+C=2$ $\quad$ $4A+4B+D=-4$

$-A+C+D=0$ $\quad$ $-4A=-8$

$$\int \frac{1}{x^2+4x+5}\,dx = \int \frac{1}{\underbrace{x^2+4x+4}-4+5}\,dx$$

$$= \int \frac{1}{(x+2)^2+1}\,dx$$

$$\frac{1}{x^2+5x+4} = \frac{}{x+4} + \frac{}{x+1}$$

$$(x+4)(x+1)$$

$$\int e^x \cos x\,dx$$

$u = e^x$ $\quad dv = \cos x\,dx$

$du = e^x\,dx$ $\quad v = \sin x$

$$= e^x \sin x - \int e^x \sin x\,dx$$

$u = e^x$ $\quad dv = \sin x\,dx$

$du = e^x\,dx$ $\quad v = -\cos x$

$$= e^x \sin x - \left(-e^x \cos x + \int + e^x \cos x\,dx\right)$$

$$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

$$\frac{2\int e^x \cos x\,dx}{2} = \frac{\sim\sim}{2} + C$$

$$\int \sec x \, dx$$

$$\int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$$

$$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

$u = \sec x + \tan x$

$du = (\sec x \tan x + \sec^2 x)\, dx$

$$\int \frac{1}{\sqrt{x^2 - 9}} \, dx \quad , \quad x = 3\sec\theta$$

$dx = 3\sec\theta\tan\theta\, d\theta$

$$\int \frac{3\sec\theta \cancel{\tan\theta}\, d\theta}{\underbrace{\sqrt{9(\sec^2 x - 1)}}_{\cancel{3\tan\theta}}} = \int \sec\theta \, d\theta$$

$$\int \frac{2x-4}{\sqrt{2x-x^2}}\,dx = \int \frac{2x-2}{\sqrt{2x-x^2}}\,dx - \int \frac{2}{\sqrt{\quad}}\,dx$$

$u = 2x - x^2$

$du = (2-2x)\,dx$

$-du = (2x-2)\,dx$

$$\int \frac{du}{u}$$

complete the square

arcsin