40. $\int x \arcsin x \, dx = \frac{1}{2}x^2 \arcsin x - \int \frac{x^2 dx}{2\sqrt{1-x^2}}$

$w = 1-x^2$
$x^2 = 1-u$

$x = \sin\theta$
$dx = \cos d\theta$

$u = \arcsin x \qquad dv = x\,dx$

$du = \frac{dx}{\sqrt{1-x^2}} \qquad v = \frac{1}{2}x^2$

$* \int \frac{\sin^2\theta \cdot \cos\theta \, d\theta}{2\sqrt{1-\sin^2\theta}} = \int \frac{\sin^2\theta \cancel{\cos\theta} d\theta}{2\cancel{\cos\theta}} = \frac{1}{2}\int \sin^2\theta \, d\theta$

$= \frac{1}{2}\int \frac{1-\cos 2\theta}{2} d\theta = \int \frac{1}{4} d\theta - \int \frac{1}{4}\cos 2\theta \, d\theta$

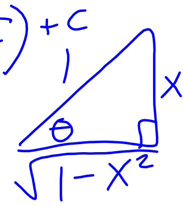
$= \frac{1}{2}x^2 \arcsin x - \left(\frac{1}{4}\theta - \left(\frac{1}{8}\sin 2\theta\right)\right) + C$

$\sin 2\theta = 2\sin\theta\cos\theta$

$x = \sin\theta$

$\arcsin x = \theta$

$= \frac{1}{2}x^2 \arcsin x - \frac{1}{4}\arcsin x + \frac{1}{8}\left(2 \cdot x \cdot \sqrt{1-x^2}\right) + C$



$= \boxed{\frac{1}{2}x^2 \arcsin x - \frac{1}{4}\arcsin x + \frac{x\sqrt{1-x^2}}{4} + C}$

44. $\int \frac{\sec^2 x}{\tan x(\tan x + 1)} dx$

$u = \tan x$
$du = \sec^2 x \, dx$

$= \int \frac{1}{u(u+1)} du = \int \left(\frac{A}{u} + \frac{B}{u+1}\right) du$

$\frac{A(u+1)}{u(u+1)} + \frac{B}{(u+1)} \cdot \frac{u}{u} = \frac{Au + A + Bu}{u(u+1)} = \frac{(A+B)u + A}{u(u+1)} = \frac{0u+1}{u(u+1)}$

$A+B = 0$
$A = 1 \Rightarrow B = -1$

$= \int \frac{1}{u} du + \int \frac{-1}{u+1} du = \ln|u| - \ln|u+1| + C$

$= \ln|\tan x| - \ln|\tan x + 1| + C$

$= \ln\left|\frac{\tan x}{\tan x + 1}\right| + C$

$\boxed{\log\frac{a}{b} = \log a - \log b}$

rewrite using partial fractions

6. $\frac{2x-1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$

$= \frac{A(x^2+1)^2 + (Bx+C)x(x^2+1) + (Dx+E)x}{x(x^2+1)^2}$

$= \frac{A(x^4+2x^2+1) + (Bx+C)(x^3+x) + (Dx^2+Ex)}{x(x^2+1)^2}$

$= \frac{\cancel{Ax^4} + \cancel{2Ax^2} + A + \cancel{Bx^4} + \cancel{Bx^2} + \cancel{Cx^3} + \cancel{Cx} + \cancel{Dx^2} + \cancel{Ex}}{x(x^2+1)^2}$

$\frac{2x-1}{x(x^2+1)^2} = \frac{(A+B)x^4 + Cx^3 + (2A+B+D)x^2 + (C+E)x + A}{x(x^2+1)^2}$

$A+B=0$ $\quad 2A+B+D=0$

$C=0$ $\quad C+E=2$

$A=-1$ $\quad -2+1+D=0$

$B=1$ $\quad E=-2$ $\quad D=1$

$\frac{2x-1}{x(x^2+1)^2} = \frac{-1}{x} + \frac{x}{x^2+1} + \frac{x+2}{(x^2+1)^2}$

5.7 Ex 3 - Find the general solution.

$(x^2+4)\frac{dy}{dx} = xy$

$\int \frac{dy}{y} = \int \frac{x\,dx}{x^2+4}$ $\qquad u=x^2+4$, $du=2x\,dx$, $\frac{1}{2}du = x\,dx$

$\ln|y| = \int \frac{1}{2}\frac{du}{u}$

$\ln|y| = \frac{1}{2}\ln(x^2+4) + C$

$e^{\ln|y|} = e^{\frac{1}{2}\ln(x^2+4)+C}$ $\qquad x^{m+n} = x^m x^n$

$|y| = e^{\ln\sqrt{x^2+4}}\, e^{C}$ $\quad e^C = a$ $\qquad a\log_b x = \log_b x^a$

$|y| = a\sqrt{x^2+4}$

$\boxed{y = \pm a\sqrt{x^2+4}}$ $\leftarrow$ general solution