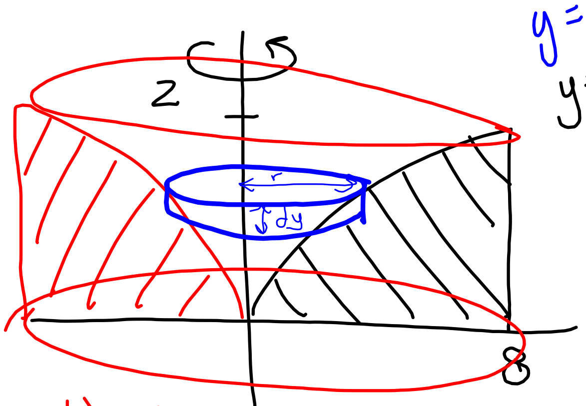

$y=f(x)$

$y=\sqrt[3]{x}$ / $y^3=x$

$f(y)=x$

outer cylinder − inner curvy cone

$\pi(8)^2\cdot 2 - \int_0^2 \pi(y^3)^2\,dy$

$\int_0^2 \pi\cdot 8^2\,dy$

$$\int \frac{1}{x^2+6x+13}\,dx$$

$$=\int \frac{1}{\underbrace{x^2+6x+9}+4}\,dx$$

$$=\int \frac{1}{(x+3)^2+2^2}\,dx \qquad u=x+3,\ a=2$$

$$=\frac{1}{2}\arctan\frac{x+3}{2}+C$$

$$\int \frac{du}{u^2+a^2}$$

u ↑ function, a ↑ constant

$$=\frac{1}{a}\arctan\frac{u}{a}+C$$

$(a+b)^2=a^2+2ab+b^2$

$(x+a)^2=x^2+2ax+a^2$

$$\int \frac{1}{x^2-7x+6}\,dx = \int \frac{dx}{(x-6)(x-1)}$$

$$= \int\left(\frac{A}{x-6}+\frac{B}{x-1}\right)dx$$

$$\frac{A(x-1)+B(x-6)}{(x-6)(x-1)} \Rightarrow Ax-A+Bx-6B$$

$$= (A+B)x+(-A-6B)$$

$$\frac{(A+B)x+(-A-6B)}{(x-6)(x-1)} = \frac{1}{(x-6)(x-1)}$$

$$\begin{cases} A+B=0 \\ -A-6B=1 \end{cases} \qquad A=-B$$

$$0-5B=1 \qquad A=1/5$$

$$B=-1/5$$

$$= \int\left(\frac{A}{x-6}+\frac{B}{x-1}\right)dx = \int \frac{1}{5}\cdot\frac{1}{x-6}\,dx + \int\left(\frac{-1}{5}\right)\frac{1}{x-1}\,dx$$

$$= \boxed{\frac{1}{5}\ln|x-6| - \frac{1}{5}\ln|x-1| + C}$$

$$= \ln\left(|x-6|\right)^{1/5} - \ln\left(|x-1|\right)^{1/5} + C$$

$$= \ln\left|\frac{x-6}{x-1}\right|^{1/5} + C$$

$$\int \cos^2 6x\,dx$$

$$= \int\left(\frac{1}{2}\cos 12x + \frac{1}{2}\right)dx$$

$$= \frac{1}{2}\,\frac{\sin 12x}{12} + \frac{1}{2}x$$

$$= \frac{1}{24}\sin 12x + \frac{1}{2}x + C$$

$$= \frac{1}{24}\left(2\sin 6x\cos 6x\right) + \frac{1}{2}x + C$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta + 1 = 2\cos^2\theta$$

$$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$$

$$\frac{\cos 12x}{2} + \frac{1}{2} = \cos^2 6x$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\sin 2(6x) = 2\sin 6x\cos 6x$$

$$\int x\sin^2 x\,dx$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$= \int x\left(\frac{1-\cos 2x}{2}\right)dx = \int \frac{x\cdot 1 - x\cos 2x}{2}\,dx$$

$$= \int \left(\frac{x}{2} - \frac{x\cos 2x}{2}\right)dx$$

$$= \int \frac{x}{2}\,dx - \int \frac{1}{2}x\cos 2x\,dx$$

$$= \frac{1}{4}x^2 - \left(\frac{1}{4}x\sin 2x - \int \frac{1}{4}\sin 2x\,dx\right)$$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$u = \frac{1}{2}x \quad dv = \cos 2x\,dx$

$du = \frac{1}{2}dx \quad v = \frac{1}{2}\sin 2x$

$$= \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x + \int \frac{1}{4}\sin 2x\,dx$$

$$= \boxed{\frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + C}$$

$$\int e^{8x}\sin 9x\,dx$$

$u = e^{8x} \quad dv = \sin 9x\,dx$

$du = 8e^{8x}dx \quad v = -\frac{1}{9}\cos 9x$

$$= -\frac{1}{9}e^{8x}\cos 9x + \int \frac{8}{9}e^{8x}\cos 9x\,dx$$

$u = \frac{8}{9}e^{8x} \quad dv = \cos 9x\,dx$

$du = \frac{64}{9}e^{8x}dx \quad v = \frac{1}{9}\sin 9x$

$$\int e^{8x}\sin 9x\,dx$$

$$= -\frac{1}{9}e^{8x}\cos 9x + \frac{8}{81}e^{8x}\sin 9x - \int \frac{64}{81}e^{8x}\sin 9x\,dx$$

$$\frac{81+64}{81}\int e^{8x}\sin 9x\,dx = -\frac{1}{9}e^{8x}\cos 9x + \frac{8}{81}e^{8x}\sin 9x$$

$$\int e^{8x}\sin 9x\,dx = \frac{81}{145}\left(-\frac{1}{9}e^{8x}\cos 9x + \frac{8}{81}e^{8x}\sin 9x\right) + C$$