$\int \frac{4}{x^2+9}\,dx = \frac{4}{3}\arctan\frac{x}{3}+C$

$\int \frac{4x}{x^2+9}\,dx$

$u = x^2+9$

$du = 2x\,dx$

$2du = 4x\,dx$

$\int \frac{2\,du}{u}$

$= 2\ln|u| + C$

$= \boxed{2\ln(x^2+9)+C}$

$\int \frac{4x^2}{x^2+9}\,dx$

$x^2+9 \overline{)\,4x^2}$ quotient 4, remainder -36

$\int \left(4 - \frac{36}{x^2+9}\right)dx$

$= \int 4\,dx - \int \frac{36}{x^2+9}\,dx$

$= \boxed{4x - 12\arctan\frac{x}{3}+C}$

Integration by parts

$\int u\,dv = uv - \int v\,du$

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12. $\int \frac{5x}{e^{2x}}\,dx = \int 5xe^{-2x}\,dx$

$u = 5x$; $dv = e^{-2x}\,dx$

$du = 5dx$; $v = \int e^{-2x}\,dx$ $\quad u=2x,\ du=-2dx,\ -\frac{1}{2}du=dx$

$v = -\frac{1}{2}e^{-2x}$

$\int \frac{5x}{e^{2x}}\,dx = -\frac{5}{2}xe^{-2x} + \int +\frac{5}{2}e^{-2x}\,dx$

$= \boxed{-\frac{5}{2}xe^{-2x} - \frac{5}{4}e^{-2x} + C}$

30. $\int e^{4x}\cos 2x\,dx$

$u = e^{4x}$ $\quad dv = \cos 2x\,dx$ $\quad u=2x,\ du=2dx,\ \frac{1}{2}du=dx$

$du = 4e^{4x}dx$ $\quad v = \frac{1}{2}\sin 2x$

$\int e^{4x}\cos 2x\,dx = \frac{1}{2}e^{4x}\sin 2x - \int 2e^{4x}\sin 2x\,dx$

$u = 2e^{4x}$ $\quad dv = \sin 2x\,dx$

$du = 8e^{4x}dx$ $\quad v = -\frac{1}{2}\cos 2x$

$\int e^{4x}\cos 2x\,dx = \frac{1}{2}e^{4x}\sin 2x - \left(-e^{4x}\cos 2x - \int -4e^{4x}\cos 2x\,dx\right)$

$\int e^{4x}\cos 2x\,dx = \frac{1}{2}e^{4x}\sin 2x + e^{4x}\cos 2x - 4\int e^{4x}\cos 2x\,dx$

$\int e^{4x}\cos 2x\,dx = \frac{\frac{1}{2}e^{4x}\sin 2x + e^{4x}\cos 2x}{5} + C$

$= \boxed{\frac{1}{10}e^{4x}\sin 2x + \frac{1}{5}e^{4x}\cos 2x + C}$

8.3 Trigonometric Integrals

2. $\int \cos^3 x \sin^4 x\,dx$

$\int \cos^2 x \sin^4 x \cos x\,dx$

$= \int (1-\sin^2 x)\sin^4 x \cos x\,dx$

$= \int (\sin^4 x - \sin^6 x)\cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int (u^4 - u^6)\,du = \frac{1}{5}u^5 - \frac{1}{7}u^7 = \boxed{\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\frac{1+\cos 2x}{2} = \cos^2 x$; $\sin^2 x = \frac{1-\cos 2x}{2}$

10. $\int \sin^4 6\theta\,d\theta = \int (\sin^2 6\theta)^2\,d\theta$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$= \int \left(\frac{1-\cos 12\theta}{2}\right)^2 dx$

$= \frac{1}{4}\int (1 - 2\cos 12\theta + \cos^2 12\theta)\,d\theta$

$= \frac{1}{4}\int \left(1 - 2\cos 12\theta + \frac{1+\cos 24\theta}{2}\right)d\theta$

$= \frac{1}{4}\int \left(\frac{3}{2} - 2\cos 12\theta + \frac{1}{2}\cos 24\theta\right)d\theta$

$= \boxed{\frac{3}{8}\theta - \frac{1}{24}\sin 12\theta + \frac{1}{192}\sin 24\theta + C}$