

8.4 Trig Substitution

22. $\int \frac{x^2}{\sqrt{36-x^2}} dx$

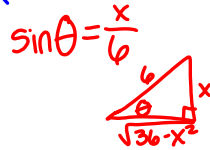
$x = 6\sin\theta$
 $x^2 = 36\sin^2\theta$
 $dx = 6\cos\theta d\theta$

$\sin^2 x + \cos^2 x = 1$
 $\tan^2 x + 1 = \sec^2 x$
 $\rightarrow \cos^2 x = 1 - \sin^2 x$

$\int \frac{36\sin^2\theta \cdot 6\cos\theta d\theta}{\sqrt{36-36\sin^2\theta} \sqrt{36(1-\sin^2\theta)}} = \int \frac{36\sin^2\theta \cdot 6\cos\theta d\theta}{\sqrt{36\cos^2\theta}}$

$\sin\theta = \frac{-\cos 2\theta}{2}$
 $= \int 36\sin^2\theta d\theta = 36 \int (\frac{1}{2} - \frac{1}{2}\cos 2\theta) d\theta$

$= 18\theta - 9\sin 2\theta + C$



$= 18\theta - 9 \cdot 2\sin\theta\cos\theta + C$

$= 18\arcsin\frac{x}{6} - 18 \cdot \frac{x}{6} \cdot \frac{\sqrt{36-x^2}}{6} + C$

$= 18\arcsin\frac{x}{6} - \frac{x\sqrt{36-x^2}}{2} + C$

40. $\frac{1}{2} \int \frac{2x-6+6}{\sqrt{x^2-6x+5}} dx = \frac{1}{2} \int \frac{2x-6}{\sqrt{x^2-6x+5}} dx + \frac{1}{2} \int \frac{6}{\sqrt{x^2-6x+5}} dx$

$u = x^2 - 6x + 5$
 $du = (2x-6)dx$

$= \frac{1}{2} \int u^{-1/2} du + \int \frac{3}{\sqrt{x^2-6x+5}} dx$

$= \sqrt{x^2-6x+5} + \int \frac{3}{\sqrt{x^2-6x+9-9+5}} dx$

$= \sqrt{x^2-6x+5} + \int \frac{3}{\sqrt{(x-3)^2-4}} dx$

$x-3 = 2\sec\theta$
 $dx = 2\sec\theta\tan\theta d\theta$

$\sin^2 x + \cos^2 x = 1$
 $\tan^2 x + 1 = \sec^2 x$
 $\tan^2 x = \sec^2 x - 1$

$= \sqrt{x^2-6x+5} + \int \frac{6\sec\theta\tan\theta d\theta}{\sqrt{4\sec^2\theta-4}}$

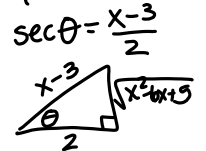
$= \sqrt{x^2-6x+5} + \int 3 \frac{\sec\theta \cdot \sec\theta + \tan\theta}{\sec\theta + \tan\theta} d\theta$

$= \sqrt{x^2-6x+5} + 3 \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\tan\theta + \sec\theta} d\theta$

$= \sqrt{x^2-6x+5} + 3 \ln|\tan\theta + \sec\theta| + C$

$u = \tan\theta + \sec\theta$
 $du = \sec^2\theta + \sec\theta\tan\theta$
 $\int \frac{du}{u} = \ln|u|$

$= \sqrt{x^2-6x+5} + 3 \ln \left| \frac{\sqrt{x^2-6x+5}}{2} + \frac{x-3}{2} \right| + C$



8.5 Partial Fractions

8. $\int \frac{3-x}{3x^2-2x-1} dx = \int \frac{3-x}{(3x+1)(x-1)} dx$

$= \int \left(\frac{A}{3x+1} + \frac{B}{x-1} \right) dx$

$\frac{A}{3x+1} \cdot \frac{x-1}{x-1} + \frac{B}{x-1} \cdot \frac{3x+1}{3x+1} = \frac{Ax-A+3Bx+B}{(3x+1)(x-1)}$

$= \frac{(A+3B)x + B-A}{(3x+1)(x-1)} = \frac{3-x}{(3x+1)(x-1)}$

$\begin{cases} A+3B = -1 \\ B-A = 3 \end{cases}$