From http://en.wikipedia.org/wiki/Orbifold_notation:

In geometry, **orbifold notation** (or **orbifold signature**) is a system, invented by William Thurston and popularized by the mathematician John Conway, for representing types of symmetry groups in two-dimensional spaces of constant curvature. The advantage of the notation is that it describes these groups in a way which indicates many of the groups' properties: in particular, it describes the orbifold obtained by taking the quotient of Euclidean space by the group under consideration.

Groups representable in this notation include the point groups on the sphere (S^2), the frieze groups and wallpaper groups of the Euclidean plane (E^2), and their analogues on the hyperbolic plane (H^2).

Symbol	Cost ($)	Symbol	Cost ($)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
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4	$\frac{3}{4}$	4	$\frac{3}{8}$
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⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.1. Costs of symbols in signatures.

Each symbol corresponds to a distinct transformation:

- an integer *n* to the left of an asterisk indicates a rotation of order *n* around a gyration point
- an integer *n* to the right of an asterisk indicates a transformation of order 2*n* which rotates around a kaleidoscopic point and reflects through a line (or plane)
- an × indicates a glide reflection
- the exceptional symbol *o* indicates that there are precisely two linearly independent translations.

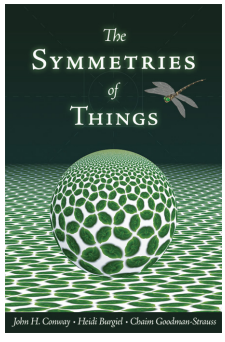
The Euler characteristic and the order [edit]

The Euler characteristic of an orbifold can be read from its Conway symbol, as follows. Each feature has a value:

- *n* without or before an asterisk counts as $\frac{n-1}{n}$
- *n* after an asterisk counts as $\frac{n-1}{2n}$
- asterisk and $\times$ count as 1
- o counts as 2.

Subtracting the sum of these values from 2 gives the Euler characteristic.

If the sum of the feature values is 2, the order is infinite, i.e., the notation represents a wallpaper group or a frieze group. Indeed, Conway's "Magic Theorem" indicates that the 17 wallpaper groups are exactly those with the sum of the feature values equal to 2. Otherwise, the order is 2 divided by the Euler characteristic.



17 wallpaper groups[2]

Orbifold Signature	Coxeter	Hermann–Mauguin	Speiser Niggli	Polya Guggenhein	Fejes Toth Cadwell
*632	[6,3]	p6m	$C^{(I)}_{6v}$	D_6	W^1_6
632	$[6,3]^+$	p6	$C^{(I)}_6$	C_6	W_6
*442	[4,4]	p4m	$C^{(I)}_4$	D^*_4	W^1_4
4*2	$[4^+,4]$	p4g	C^{II}_{4v}	D^o_4	W^2_4
442	$[4,4]^+$	p4	$C^{(I)}_4$	C_4	W_4
*333	$[3^{[3]}]$	p3m1	C^{II}_{3v}	D^*_3	W^1_3
3*3	$[3^+,6]$	p31m	C^I_{3v}	D^o_3	W^2_3
333	$[3^{[3]}]^+$	p3	C^I_3	C_3	W_3
*2222	[∞,2,∞]	pmm	C^I_{2v}	D_2kkkk	W^2_2
2*22	$[\infty,2^+,\infty]$	cmm	C^{IV}_{2v}	D_2kgkg	W^1_2
22*	$[(\infty,2)^+,\infty]$	pmg	C^{III}_{2v}	D_2kkgg	W^3_2
22×	$[\infty^+,2^+,\infty^+]$	pgg	C^{II}_{2v}	D_2gggg	W^4_2
2222	$[\infty,2,\infty]^+$	p2	$C^{(I)}_2$	C_2	W_2
**	$[\infty^+,2,\infty]$	pm	C^I_s	D_1kk	W^2_1
*×	$[\infty^+,2^+,\infty]$	cm	C^{III}_s	D_1kg	W^1_1
××	$[\infty^+,(2,\infty)^+]$	pg	C^{II}_2	D_1gg	W^3_1
o	$[\infty^+,2,\infty^+]$	p1	$C^{(I)}_1$	C_1	W_1

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Table 3.1. Costs of symbols in signatures.

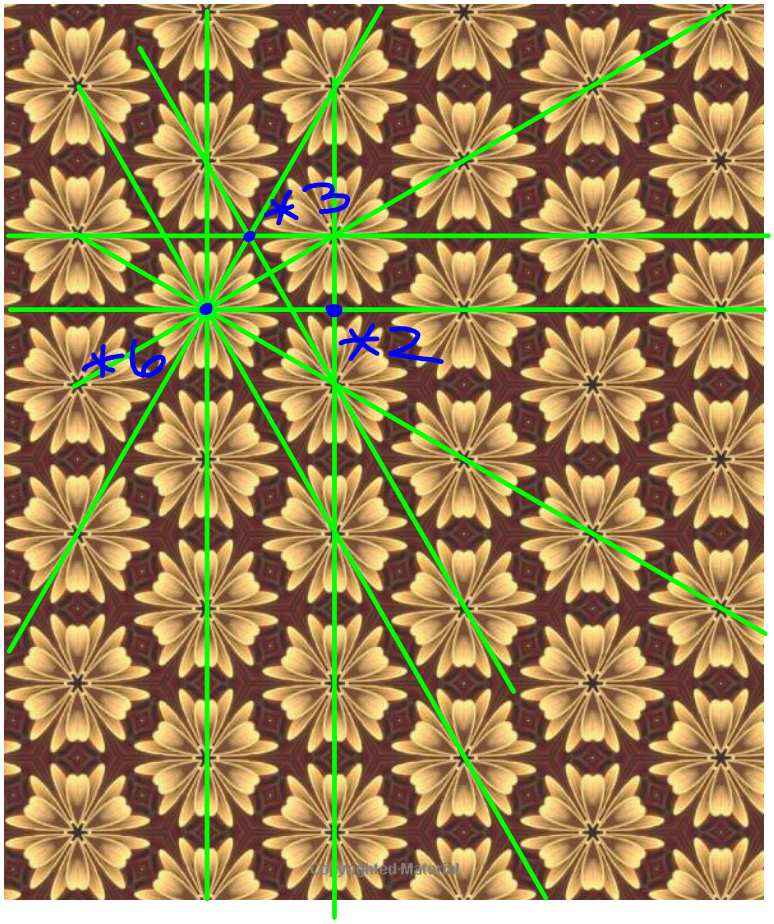

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Table 3.1. Costs of symbols in signatures.

*632

$1+\frac{5}{12}+\frac{1}{3}+\frac{1}{4}$

$1+\frac{5}{12}+\frac{4}{12}+\frac{3}{12}=2$

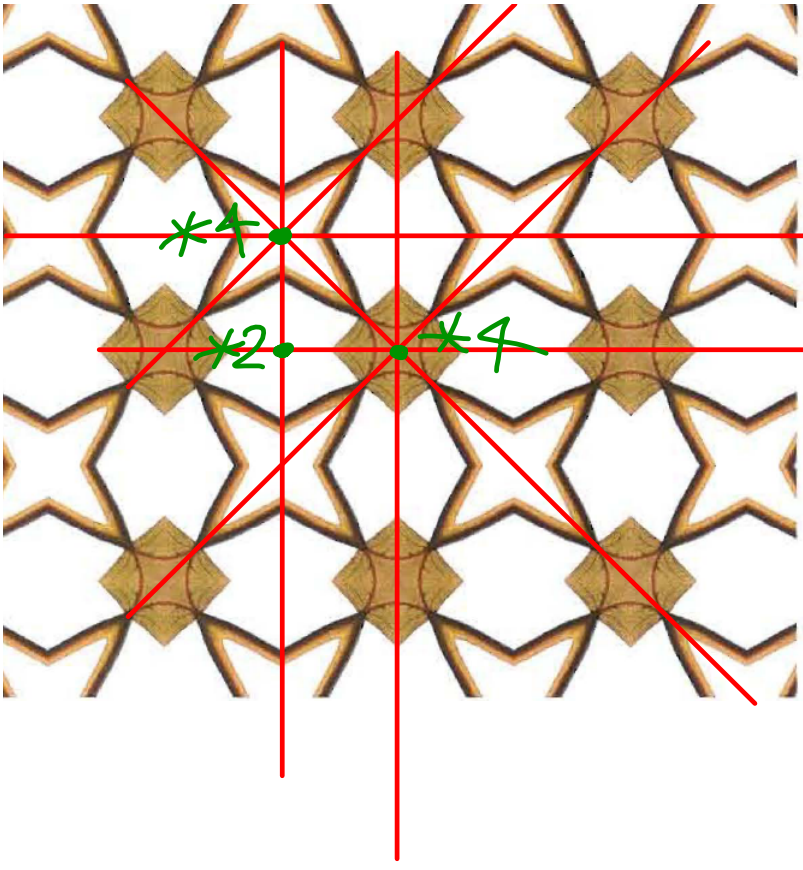

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Table 3.1. Costs of symbols in signatures.

*442

$1+\frac{3}{8}+\frac{3}{8}+\frac{1}{4}$

$1+\frac{3}{8}+\frac{3}{8}+\frac{2}{8}=2$

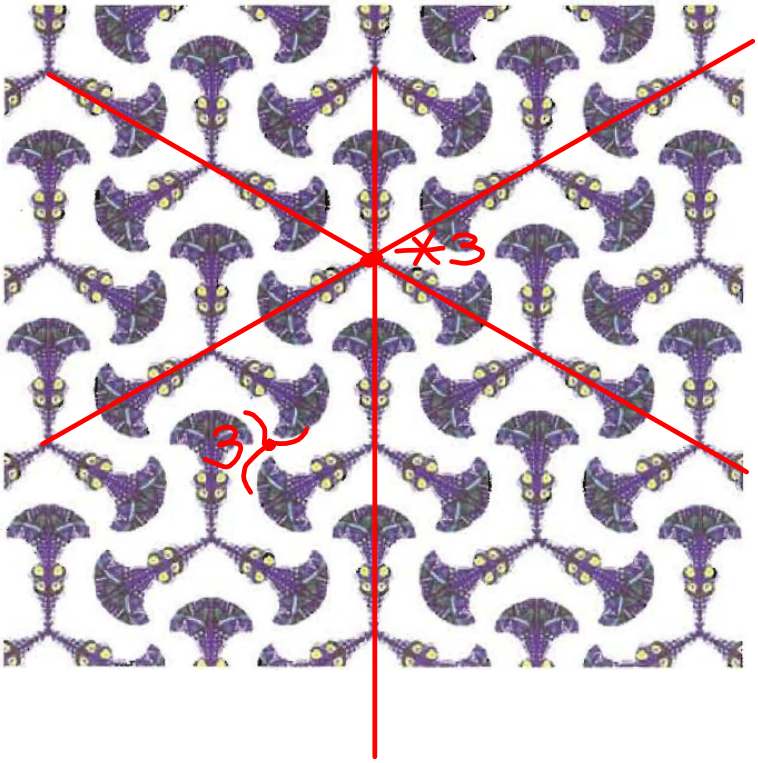

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Table 3.1. Costs of symbols in signatures.

3*3

$= \frac{2}{3} + 1 + \frac{1}{3} = 2$

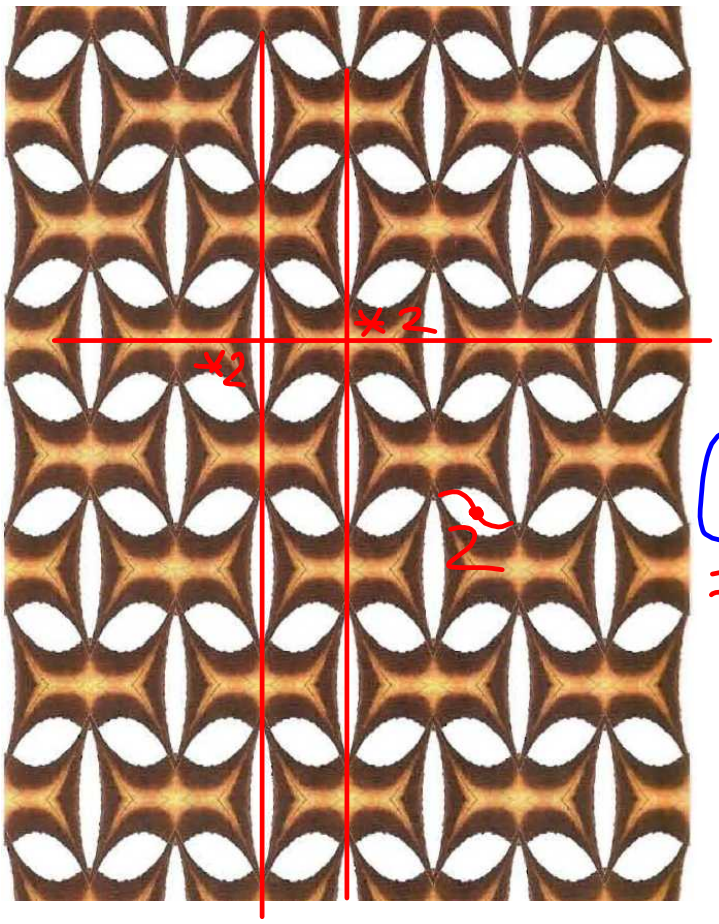

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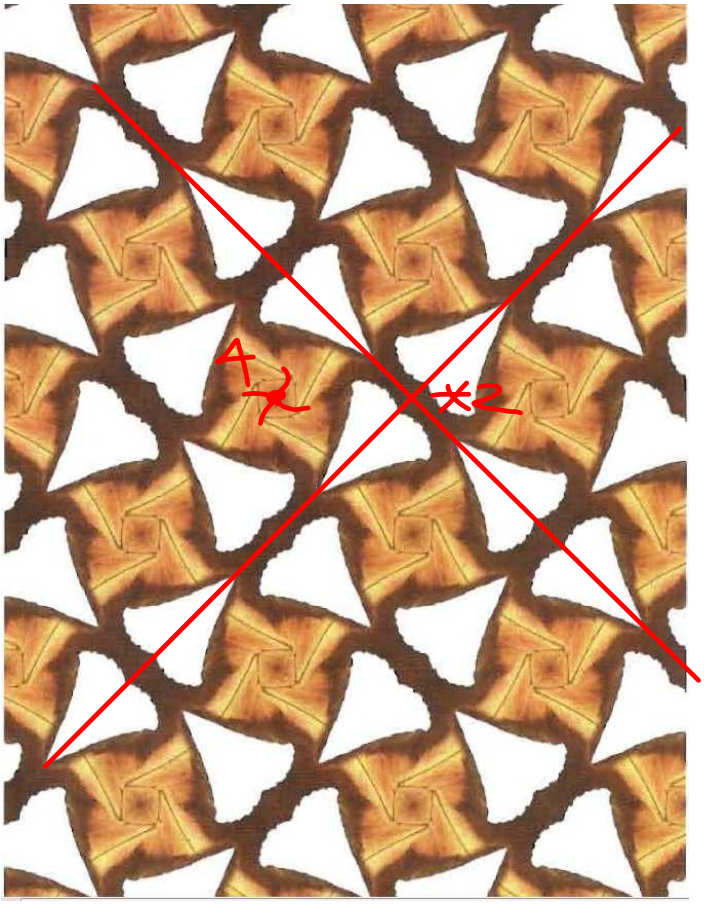
Table 3.1. Costs of symbols in signatures.

2*22

$= \frac{1}{2} + 1 + \frac{1}{4} + \frac{1}{4} = 2$



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$4*2$

$= \frac{3}{4} + 1 + \frac{1}{4} = 2$

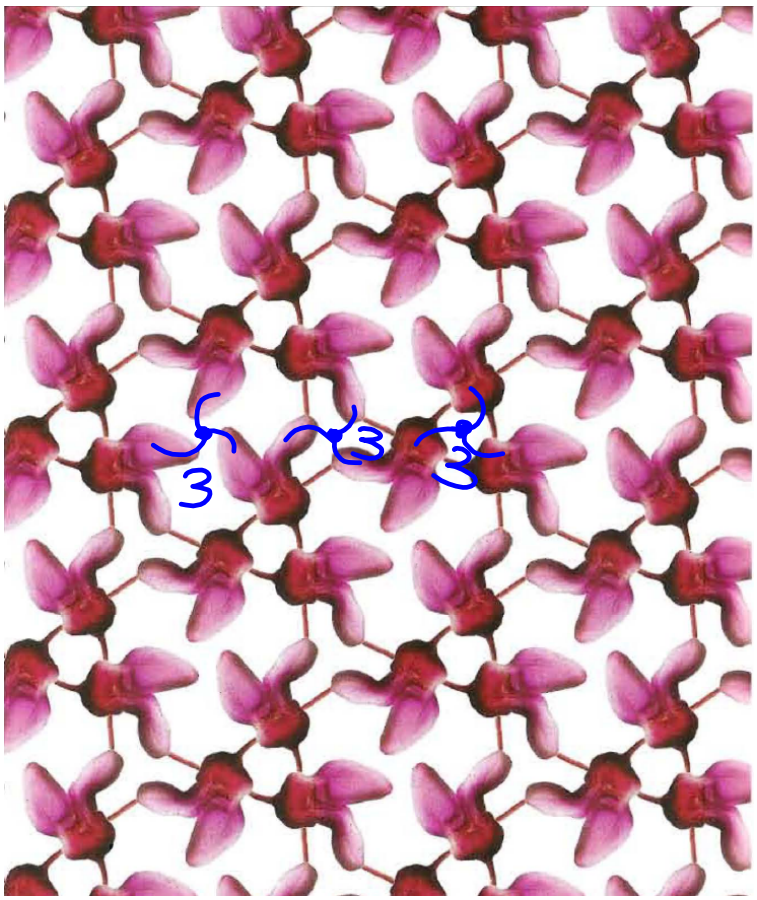

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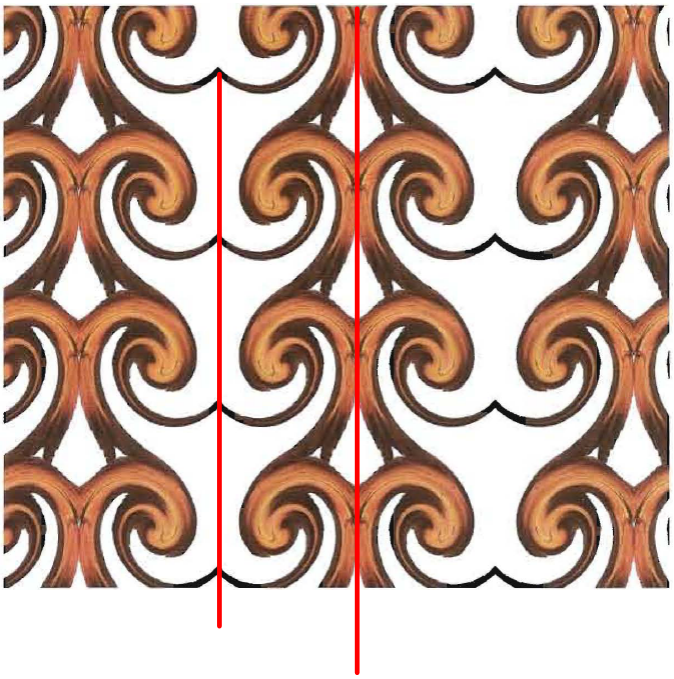
Table 3.1. Costs of symbols in signatures.

333

$= \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{6}{3} = 2$



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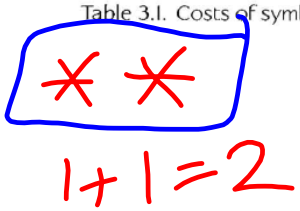
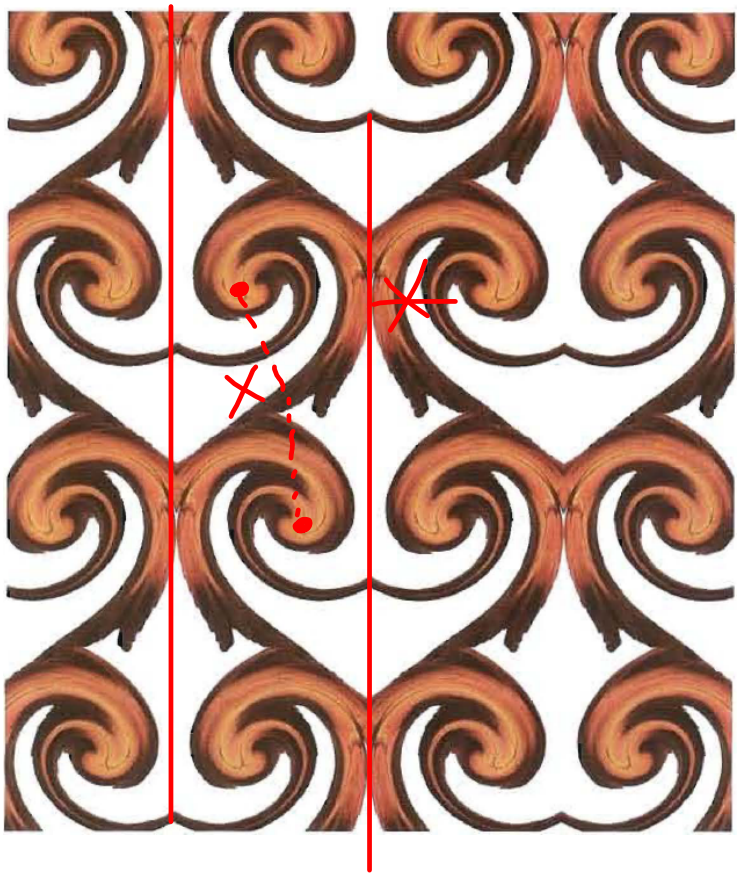


1+1=2



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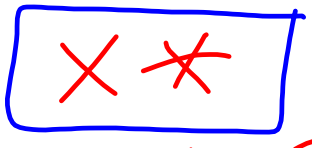

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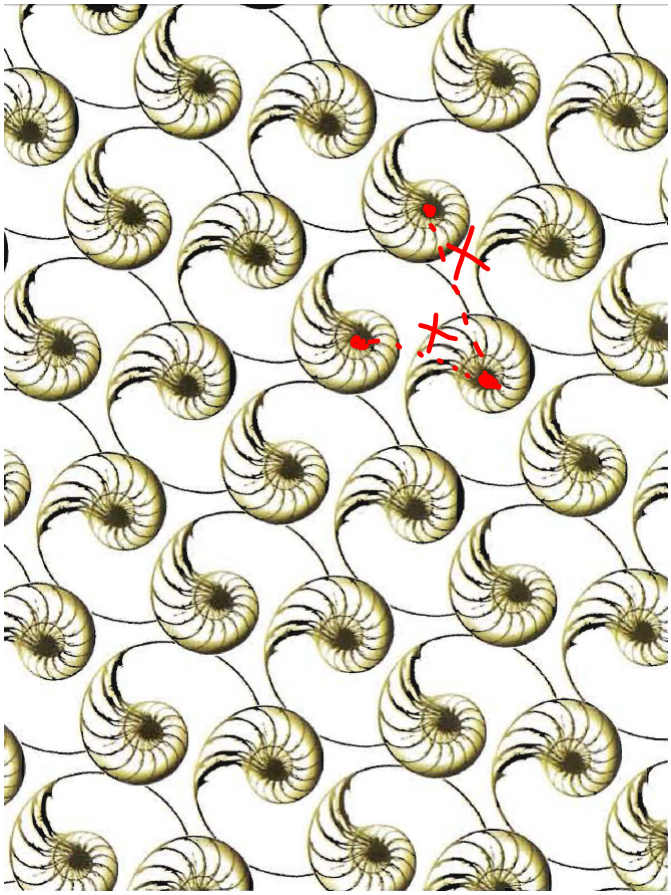

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XX

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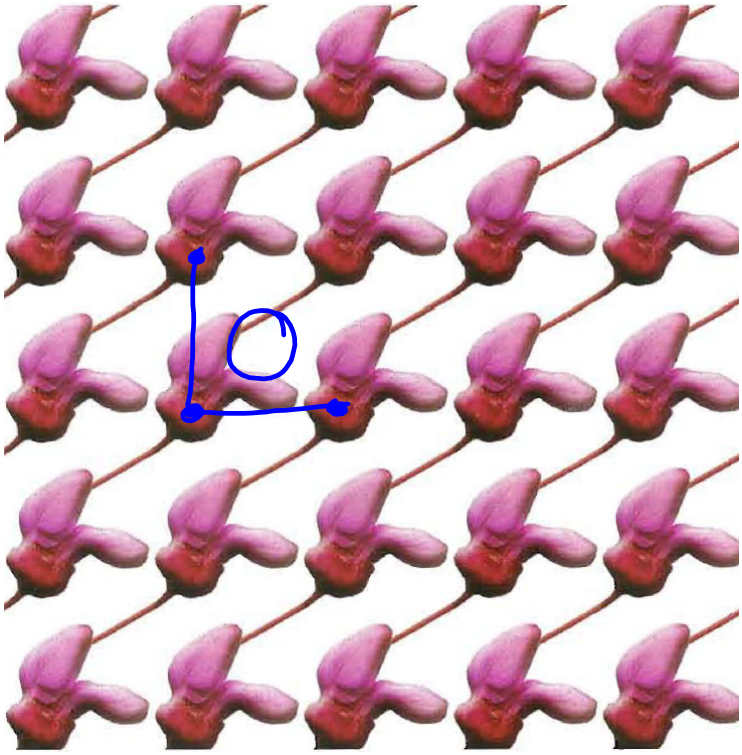

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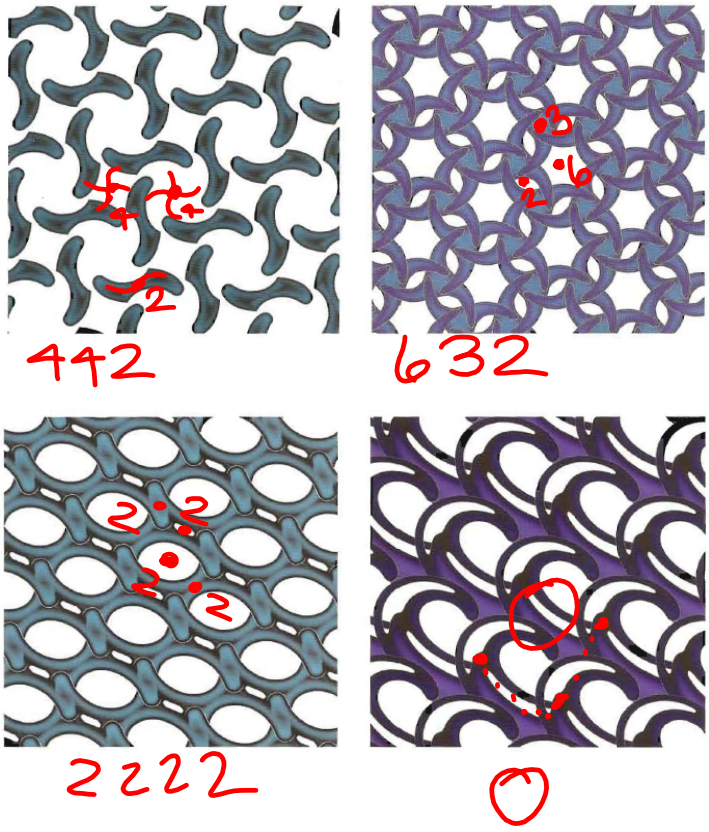
○

2



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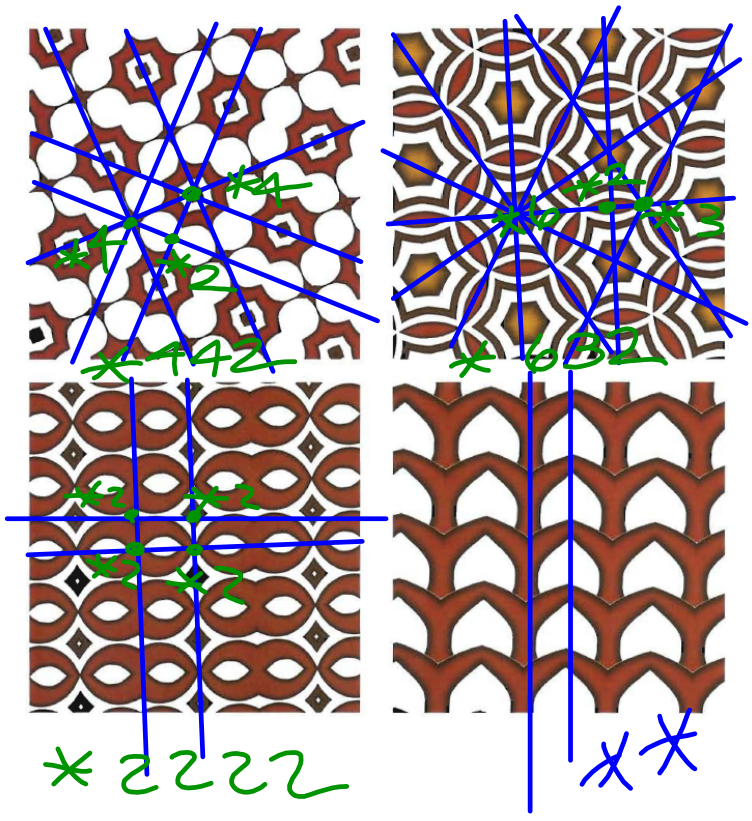

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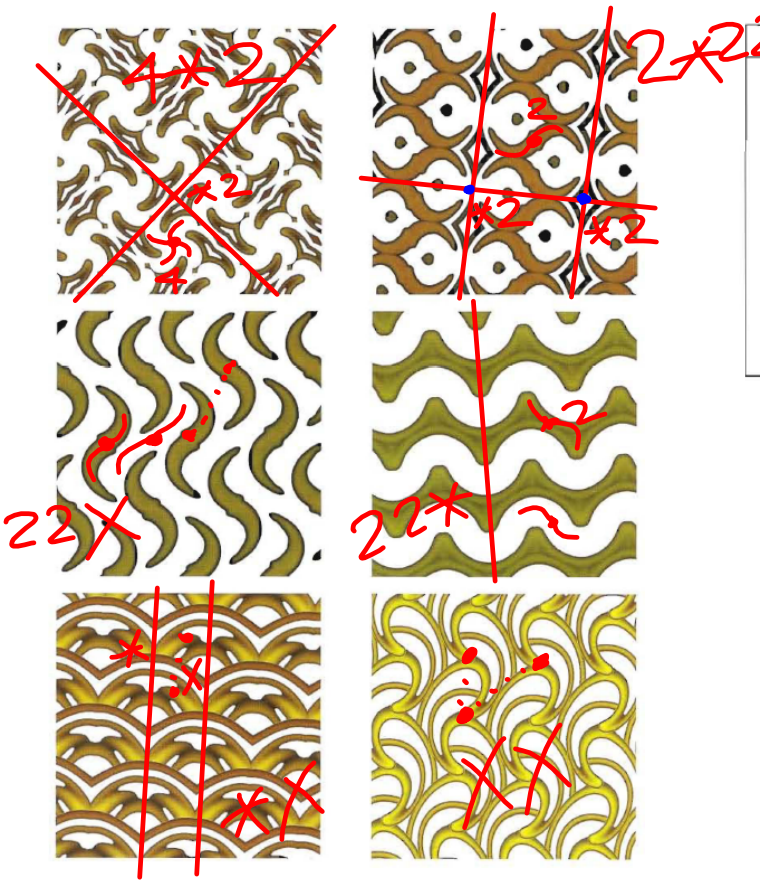

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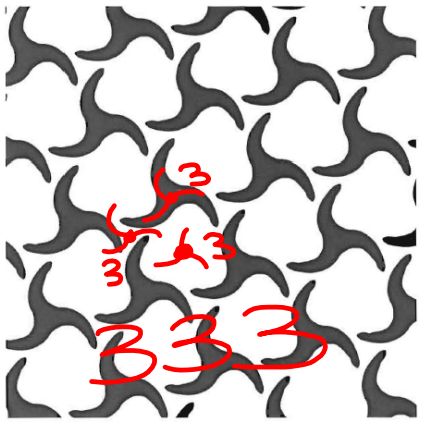

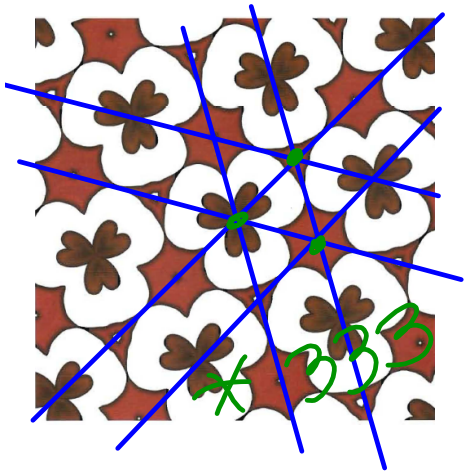

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