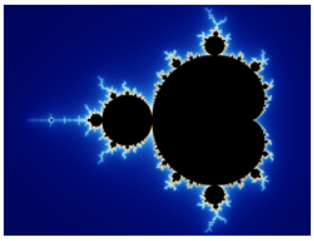
FRACTALS

Fractals:

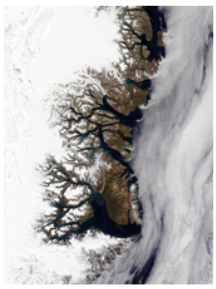
- are self-similar (at least approximately), i.e. have the rescaling property (when you zoom in on a piece it looks like the whole)
- have fine structure on arbitrarily small scales
- often have simple, recursive definitions

Mandelbrot Set



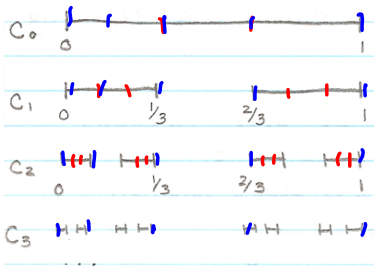
Romanesco broccoli

Coastline of Norway



$z^2 + c$

The Cantor Middle-Third Set:



C_0 has length 1

remove 1/3 $= 2^0/3^1$

C_1 has length $2/3^1$

remove 2/9 $= 2^1/3^2$

C_2 has length 4/9 $= 2^2/3^2$

remove 4/27 $= 2^2/3^3$

C_3 has length 8/27 $= 2^3/3^3$

To obtain C_{n+1} from C_n, we remove the middle third of each interval in C_n. The Cantor set C is the intersection of all C_n. C is a fractal.

C_n consists of 2^n closed intervals of length $\frac{1}{3^n}$. The total length of C_n is $\left(\frac{2}{3}\right)^n$ which approaches 0 as n approaches ∞. Hence the "length" of C is 0.

Another way to state this is that the length of $C_{n+1} = \frac{2}{3} \cdot length\ of\ C_n$. Given this recursive definition, again we have that the length of C_n is , which approaches 0 as n approaches ∞.

$3^{n+1} = 3^n \cdot 3^1$

The total length removed from the interval [0 , 1] in the construction of the Cantor set is

$$\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \cdots = \sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{1}{3}\left(\frac{2}{3}\right)^n = \frac{\frac{1}{3}}{1-\frac{2}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

Hence we have a set from which its entire length has been removed. Yet there are still infinitely many points left in the set. Which points are they?

the endpoints of all the intervals!

Note: the sum of an infinite geometric series with common ratio less than 1 in absolute value is equal to $S_\infty = \frac{a_1}{1-r}$, where a_1 is the first term and r is the common ratio.

Homework:

- Register for turnitin.com by WEDNESDAY, 8/13
  - > Class ID: 8299378
  - > Enrollment Password: vismath
- Start doing research for one-page paper on Fractals - due FRIDAY, 8/15
  - > watch NOVA documentary
  - > look through articles in Google Drive folder
  - > browse artists on Bridges web site
  - > look at books in library & S201
- Work on Fractal written problems - due MONDAY, 8/18