Sierpinski Triangle:

T_0 T_1 T_2 T_3 T_4

Let T_n be the nth iteration of the Sierpinski triangle;
let A_n be the area of the nth iteration of the Sierpinski triangle; let $A_0 = 1$;
let P_n be the perimeter of the nth iteration of the Sierpinski triangle.

T_n consists of 3^n triangles of area $\left(\frac{1}{4}\right)^n$;

How much was removed?

We remove 3^n triangles of area $\left(\frac{1}{4}\right)^{n+1}$ to get T_{n+1} from T_n.

Hence the total area removed is:

$$\sum_{n=0}^{\infty} \frac{1}{4}\left(\frac{3}{4}\right)^n = \frac{\frac{1}{4}}{1-\frac{3}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$$

$P_{n+1} = \frac{3}{2} P_n$, which implies that $P_n = \left(\frac{3}{2}\right)^n$,
which approaches ∞ as n approaches ∞.
The Sierpinski triangle has zero area but an infinite perimeter!

Fractal Dimension: $S = 2$, $N = 3$, $d = \frac{\log 3}{\log 2} \approx 1.58$

	T_0	T_1
	T_0	T_1
Perimeter of this iteration	$P_0 = \frac{6}{\sqrt[4]{3}}$	$P_1 = \frac{9}{\sqrt[4]{3}}$
Perimeter of this iteration divided by perimeter of previous iteration	N/A	$\frac{P_1}{P_0} = \frac{3}{2}$
Area removed from previous iteration to obtain this iteration	N/A	$1/4$
Total area removed up to this point	0	$1/4$
Number of triangles in this iteration	1	3
Area of a triangle in this iteration	1	$1/4$
Total area of this iteration	$A_0 = 1$	$A_1 = 3/4$

T_0 T_1 T_2 T_3 T_4 T_n

	T_0	T_1	T_2	T_3	T_4
Perimeter of this iteration	$P_0 = \frac{6}{\sqrt[4]{3}}$	$P_1 = \frac{9}{\sqrt[4]{3}}$	$P_2 =$	$P_3 =$	$P_4 =$
Perimeter of this iteration divided by perimeter of previous iteration	N/A	$\frac{P_1}{P_0} = \frac{3}{2}$	$\frac{P_2}{P_1} =$	$\frac{P_3}{P_2} =$	$\frac{P_4}{P_3} =$
Area removed from previous iteration to obtain this iteration	N/A	$\frac{1}{4} = \frac{3^0}{4^1}$	$\frac{3}{16} = \frac{3^1}{4^2}$	$\frac{9}{64} = \frac{3^2}{4^3}$	$3^3/4^4$ $\frac{3^{n-1}}{4^n}$
Total area removed up to this point	0	$1/4$	$\frac{3^0}{4^1} + \frac{3^1}{4^2}$	$\frac{3^0}{4^1} + \frac{3^1}{4^2} + \frac{3^2}{4^3}$	$\sum_{i=0}^{3} \frac{3^i}{4^{i+1}}$ $\sum_{i=0}^{n-1} \frac{3^i}{4^{i+1}}$
Number of triangles in this iteration	1	3	$9 = 3^2$	$27 = 3^3$	$81 = 3^4$ 3^n
Area of a triangle in this iteration	1	$1/4$	$\frac{1}{16} = \frac{1}{4^2}$	$\left(\frac{1}{4}\right)^3$	$\left(\frac{1}{4}\right)^4$ $\left(\frac{1}{4}\right)^n$
Total area of this iteration	$A_0 = 1$	$A_1 = 3/4$	$A_2 = \frac{9}{16} = \left(\frac{3}{4}\right)^2$	$A_3 = \left(\frac{3}{4}\right)^3$	$A_4 = \left(\frac{3}{4}\right)^4$ $\left(\frac{3}{4}\right)^n$
Area of this iteration divided by area of previous iteration	N/A	$\frac{A_1}{A_0} = 3/4$	$\frac{A_2}{A_1} = 3/4$	$\frac{A_3}{A_2} = 3/4$	$\frac{A_4}{A_3} = 3/4$

The initial triangle T_0 had area $A_0 = 1$.

Total area removed up to the n^{th} iteration is $\sum_{i=0}^{n-1} \frac{3^i}{4^{i+1}}$

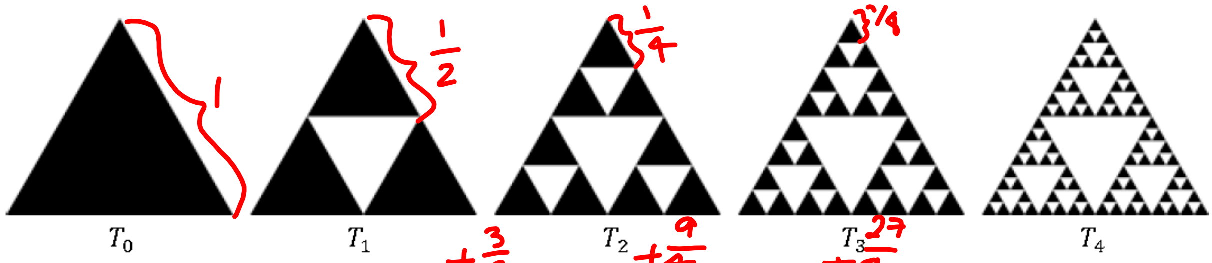
$$= \sum_{i=1}^{n} \frac{3^{i-1}}{4^i}$$

$\frac{3^{1-1}}{4^1} = \frac{3^0}{4} = \frac{1}{4}$ $\frac{3^{n-1}}{4^n}$

$\frac{3^0}{4^{0+1}} = \frac{1}{4}$ $\frac{3^{n-1}}{4^{n-1+1}} = \frac{3^{n-1}}{4^n}$

$$S_\infty = \frac{a_1}{1-r} = \frac{\frac{1}{4}}{1-\frac{3}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$$

We remove area 1



	T_0	T_1	T_2	T_3	T_4
Perimeter of this iteration	$P_0 = 3 = \frac{3^1}{2^0}$	$P_1 = \frac{9}{2} = \frac{3^2}{2^1}$	$P_2 = \frac{27}{4} = \frac{3^3}{2^2}$	$P_3 = \frac{81}{8} = \frac{3^4}{2^3}$	$P_4 = \frac{3^5}{2^4}$
Perimeter of this iteration divided by perimeter of previous iteration	N/A	$\frac{P_1}{P_0} = \frac{3}{2}$	$\frac{P_2}{P_1} = \frac{3}{2}$	$\frac{P_3}{P_2} = \frac{3}{2}$	$\frac{P_4}{P_3} = \frac{3}{2}$
Area removed from previous iteration to obtain this iteration	N/A				
Total area removed up to this point	0				
Number of triangles in this iteration	1				
Area of a triangle in this iteration	1				
Total area of this iteration	$A_0 = 1$	$A_1 =$	$A_2 =$	$A_3 =$	$A_4 =$
Area of this iteration divided by area of previous iteration	N/A	$\frac{A_1}{A_0} =$	$\frac{A_2}{A_1} =$	$\frac{A_3}{A_2} =$	$\frac{A_4}{A_3} =$

$$\boxed{P_n = \frac{3^{n+1}}{2^n}}$$

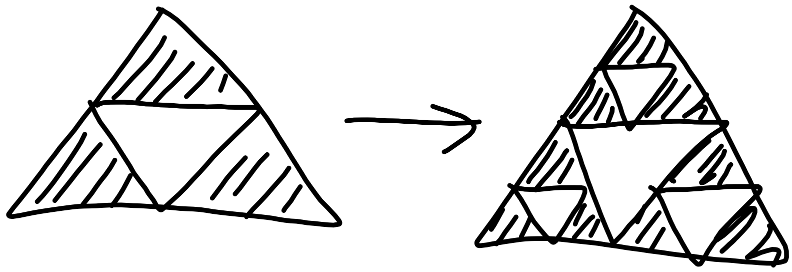
$$P_\infty = 3 + \sum_{i=1}^{\infty} \left(\frac{3}{2}\right)^n$$

$$P_\infty = 3 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \dots + \left(\frac{3}{2}\right)^n$$

$$P_n = \frac{3^{n+1}}{2^n} \longrightarrow ? \text{ as } n \to \infty$$

$$= \frac{3 \cdot 3^n}{2^n} = 3\left(\frac{3}{2}\right)^n \longrightarrow \infty$$

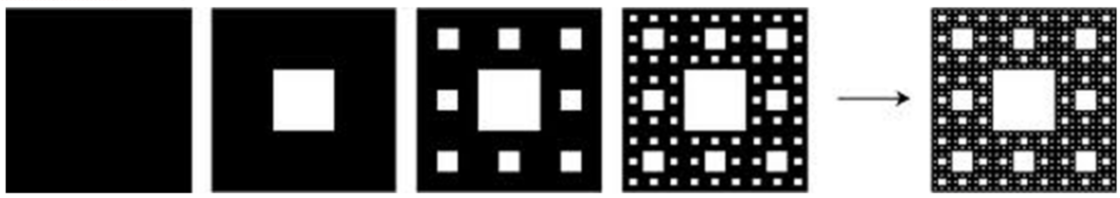
Sierpinski $\triangle$ has no area, but infinite perimeter!!



$s = 2 \; ; \; n = 3$

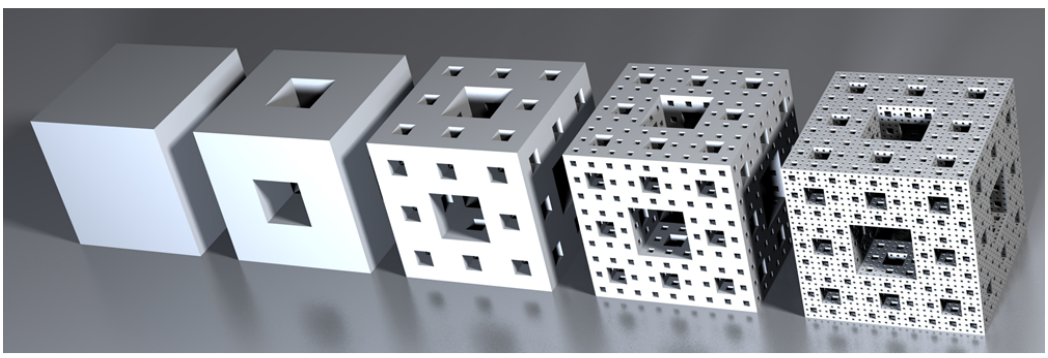
$$d = \frac{\ln n}{\ln s} = \frac{\ln 3}{\ln 2} \approx 1.58$$

Sierpinski Carpet:

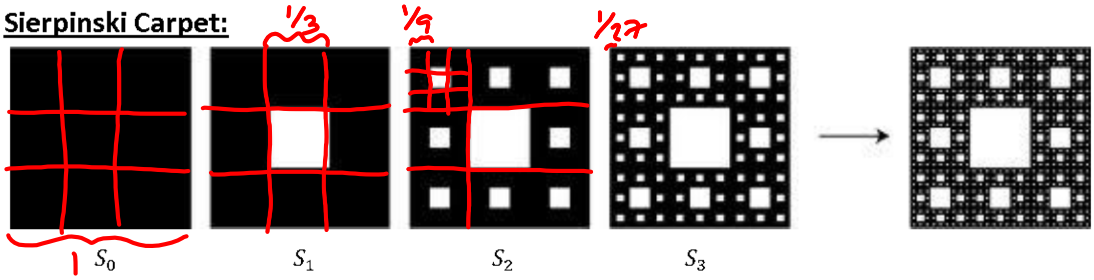


S_0 S_1 S_2 S_3

Menger Sponge:



Sierpinski Carpet:



Let S_n be the nth step of the construction of the Sierpinski Carpet. Suppose that S_0 has area 1.

	S_0	S_1	S_2	S_3
Perimeter of this iteration	$P_0 = 4$	$P_1 = 4 + \frac{4}{3}$	$P_2 = 4 + \frac{4}{3} + \frac{32}{9}$	$P_3 = P_2 + \frac{8^2 \cdot 4}{27}$
Perimeter added to previous iteration to obtain this one	N/A	$\frac{4}{3}$	$\frac{32}{9}$	$\frac{8^2 \cdot 4}{27}$
Area removed from previous iteration to obtain this iteration	N/A	$1/9 = \frac{8^0}{9^1}$	$\frac{8^1}{9^2}$	$\frac{8^2}{9^3}$
Total area removed up to this point	0	$1/9$	$1/9 + \frac{8}{9^2}$	$1/9 + 8/9^2 + 8^2/9^3$
Number of squares in this iteration	1	8	8^2	8^3
Area of a square in this iteration	1	$1/9$	$1/9^2$	$\frac{1}{(27)^2} = \frac{1}{(3^3)^2} = \frac{1}{9^3}$
Total area of this iteration	$A_0 = 1$	$A_1 = 8/9$	$A_2 = \left(\frac{8}{9}\right)^2$	$A_3 = \left(8/9\right)^3$
Area of this iteration divided by area of previous iteration	N/A	$\frac{A_1}{A_0} = 8/9$	$\frac{A_2}{A_1} = 8/9$	$\frac{A_3}{A_2} = 8/9$

Total area removed

$$A_\infty = \frac{a_1}{1-r} = \frac{1/9}{1 - 8/9} = \frac{1/9}{1/9} = 1$$

Again, we are removing all of the area.

$$4 + \frac{4}{3} + \frac{\cancel{32}\ 8^1 \cdot 4}{9} + \frac{8^2 \cdot 4}{27}$$

$$4 + \frac{4 \cdot 8^0}{3^1} + \frac{4 \cdot 8^1}{3^2} + \frac{4 \cdot 8^2}{3^3} + \dots \longrightarrow \infty$$

geometric series
common ratio : $\frac{8}{3}$

no area, infinite perimeter!

$S = 3$; $N = 8$

$$d = \frac{\ln 8}{\ln 3} \approx 1.89$$

- Register for turnitin.com by WEDNESDAY, 8/13
  - Class ID: 8299378
  - Enrollment Password: vismath
- Start doing research for one-page paper on Fractals - due FRIDAY, 8/15
  - watch NOVA documentary
  - look through articles in Brewer's Google Drive folder
  - browse articles in the searchable Bridges archive
  - look at books in library & S201
    - In the ASMS Library:
      - Benoit Mandelbrot's *Fractal Geometry of Nature*
      - Mandelbrot's *The (mis)behavior of markets: a fractal view of risk, ruin, and reward*
      - *Chaos and fractals: new frontiers of science*
    - In S201:
      - *Chaos and Fractals: The Mathematics Behind the Computer Graphics*
      - *Fractals, Chaos, Power Laws*
      - *Chaos, Fractals, and Dynamics: Computer Experiments in Mathematics*
      - *Viewpoints: Mathematical Perspective and Fractal Geometry in Art*
  - search databases in the Alabama Virtual Library
    - EBSCOhost
    - InfoTrac
    - Gale Virtual Reference Library

- Work on Fractal written problems - due MONDAY, 8/18