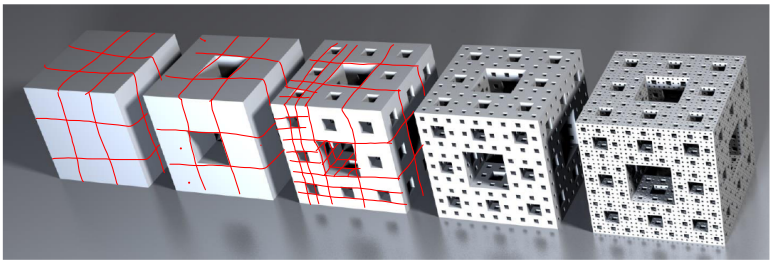
Menger Sponge:



For the **Menger Sponge**, assume that the volume of M_0 is 1.

	M_0	M_1	M_2	M_3
Surface area of this iteration	$A_0 = 6$	$A_1 = \frac{6 \cdot 12}{9} = 8$	$A_2 = \frac{6 \cdot 12^2}{9^2}$	$A_3 = \frac{6 \cdot 12^3}{9^3}$
Volume removed from previous iteration to obtain this iteration	N/A	$7 \cdot \frac{1}{27} = \frac{7}{27}$	$\frac{20 \cdot 7}{9^3} = \frac{20 \cdot 7}{27^2}$	$\frac{20^2 \cdot 7}{27^3}$
Total volume removed up to this point	0	$\frac{7}{27}$	$\frac{7}{27} + \frac{20 \cdot 7}{9^3}$	$\frac{7}{27} + \frac{20 \cdot 7}{27^2} + \frac{20^2 \cdot 7}{27^3}$
Number of cubes in this iteration	1	20	20^2	20^3
Volume of a cube in this iteration	1	$\frac{1}{27} = \left(\frac{1}{3}\right)^3$	$\left(\frac{1}{9}\right)^3 \div 9^0 = \frac{1}{27^2}$	$\left(\frac{1}{27}\right)^3$
Total volume of this iteration	$V_0 = 1$	$V_1 = \frac{20}{27}$	$V_2 = \left(\frac{20}{27}\right)^2$	$V_3 = \left(\frac{20}{27}\right)^3$
Volume of this iteration divided by volume of previous iteration	N/A	$\frac{V_1}{V_0} = \frac{20}{27}$	$\frac{V_2}{V_1} = \frac{20}{27}$	$\frac{V_3}{V_2} = \frac{20}{27}$

$$A_n = \frac{6 \cdot 12^n}{9^n} = 6 \cdot \left(\frac{12}{9}\right)^n = 6\left(\frac{4}{3}\right)^n$$

$$V_n = \left(\frac{20}{27}\right)^n$$

$\frac{7}{27} + \frac{20 \cdot 7}{27^2} + \frac{20^2 \cdot 7}{27^3}$

Total Volume removed: all of it!

$$\frac{7 \cdot 20^0}{27} + \frac{7 \cdot 20^1}{27^2} + \frac{7 \cdot 20^2}{27^3} + \cdots + \frac{7 \cdot 20^n}{27^{n+1}}$$

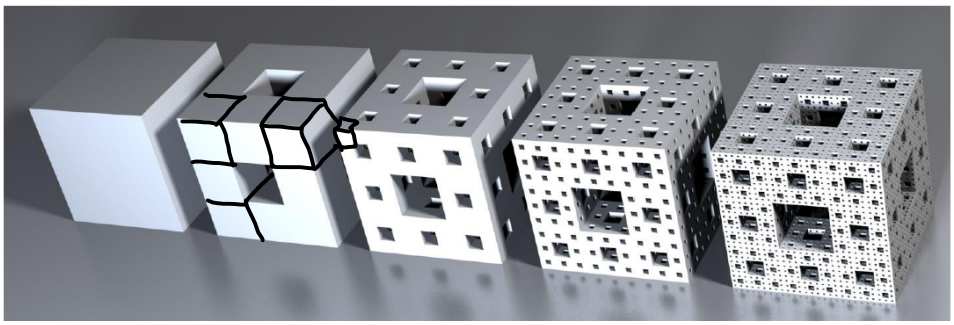
$$= \frac{7}{27}\left(\frac{20}{27}\right)^0 + \frac{7}{27}\left(\frac{20}{27}\right)^1 + \frac{7}{27}\left(\frac{20}{27}\right)^2 + \cdots + \frac{7}{27}\left(\frac{20}{27}\right)^n$$

$$\sum_{i=0}^{\infty} \frac{7}{27}\left(\frac{20}{27}\right)^i = \frac{7/27}{1 - \frac{20}{27}} = \frac{7/27}{\frac{27}{27} - \frac{20}{27}} = \frac{7/27}{7/27} = 1$$

Total Surface Area $6\left(\frac{4}{3}\right)^n \to \infty$

Menger Sponge has No Volume, but infinite surface area

Menger Sponge:



For the **Menger Sponge**, assume that the volume of M_0 is 1.

Fractal Dimension:

$S = 3$

$N = 20$

$d = \frac{\ln 20}{\ln 3} \approx 2.73$

- Fractal written problems - due MONDAY, 8/18