

Planar Symmetry & Wallpaper Groups



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There are exactly 17 so-called "wallpaper groups," which, in orbifold notation, can be classified as:

1. \circ
2. xx
3. $*x$
4. $**$
5. 632
6. $*632$
7. 333
8. $*333$
9. $3*3$
10. 442
11. $*442$
12. $4*2$
13. 2222
14. $22x$
15. 22^*
16. $*2222$
17. $2*22$

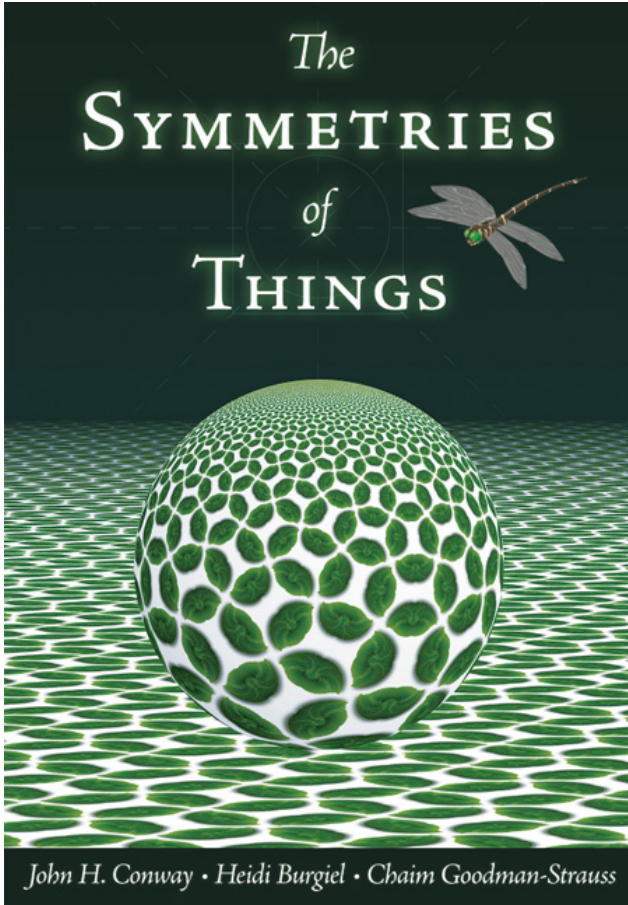
where the notations mean the following:

\circ is a "wonder ring" or "wandering" and indicates two independent directions of **translation**

x is a "miracle" or **mirrorless glide reflection**

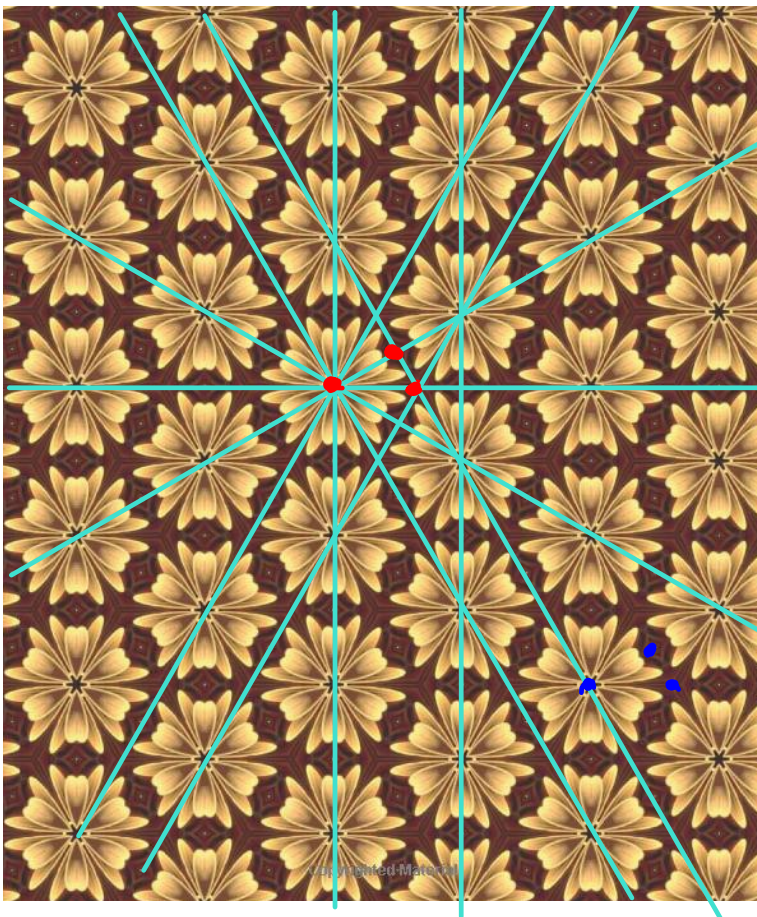
$*$ is a **mirror line**, and any numbers after a $*$ indicate different points with that many mirror lines passing through those points

numbers before a $*$ or numbers without any other notation indicate points with that number-fold **rotational symmetry**



| Symbol | Cost (\$) | Symbol | Cost (\$) |
|--------|-----------------|--------|------------------|
| ○ | 2 | * or × | 1 |
| 2 | $\frac{1}{2}$ | 2 | $\frac{1}{4}$ |
| 3 | $\frac{2}{3}$ | 3 | $\frac{1}{3}$ |
| 4 | $\frac{3}{4}$ | 4 | $\frac{3}{8}$ |
| 5 | $\frac{4}{5}$ | 5 | $\frac{2}{5}$ |
| 6 | $\frac{5}{6}$ | 6 | $\frac{5}{12}$ |
| ⋮ | ⋮ | ⋮ | ⋮ |
| N | $\frac{N-1}{N}$ | N | $\frac{N-1}{2N}$ |
| ∞ | 1 | ∞ | $\frac{1}{2}$ |

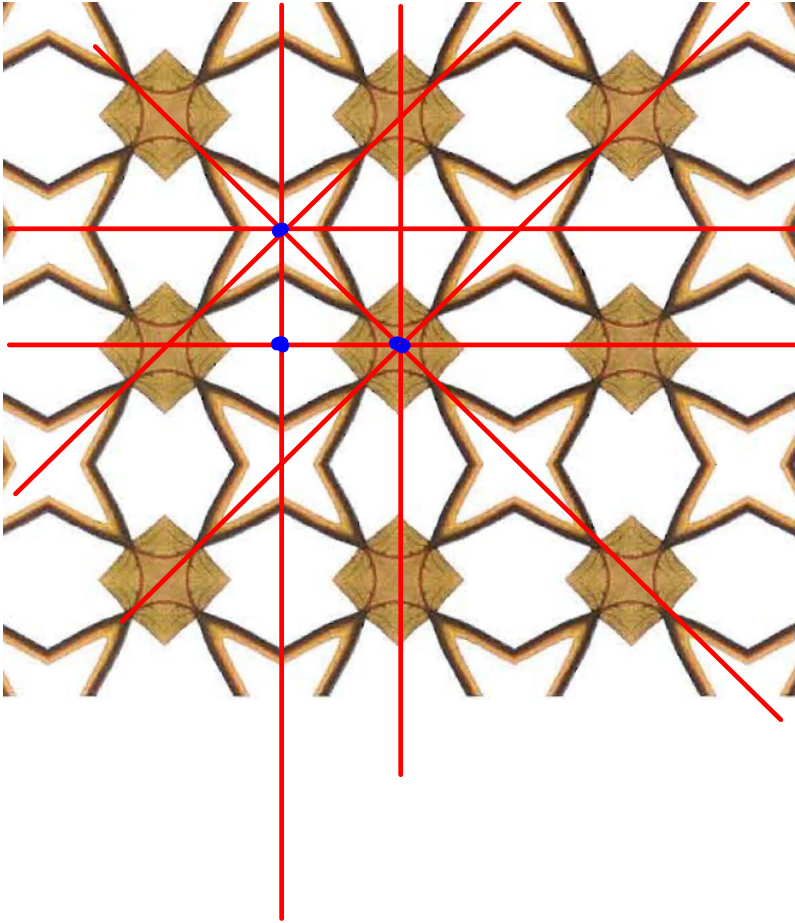
Table 3.1. Costs of symbols in signatures.



| Symbol | Cost (\$) | Symbol | Cost (\$) |
|--------|-----------------|--------|------------------|
| ○ | 2 | * or × | 1 |
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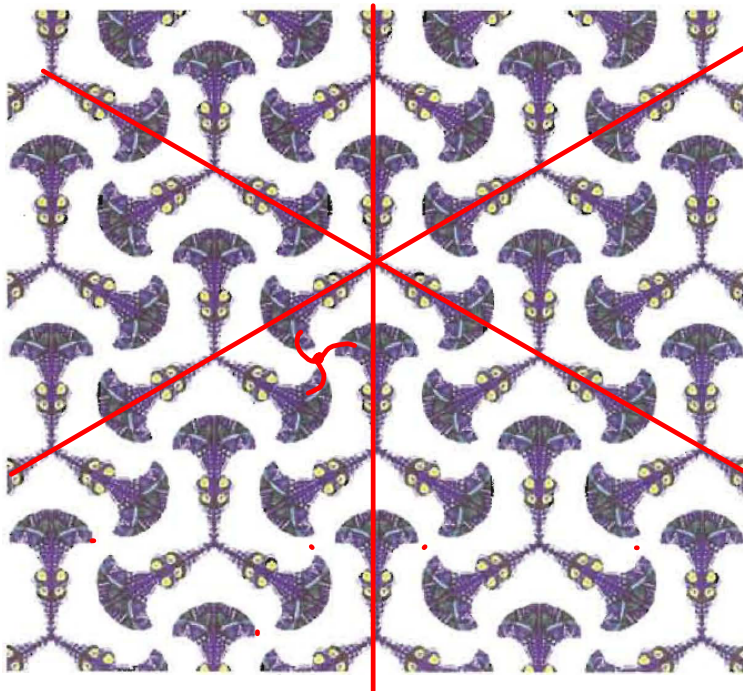
$$\begin{aligned}
 & * 632 \\
 & \$1 + \frac{5}{12} + \frac{1}{3} + \frac{1}{4} \\
 & = \frac{12}{12} + \frac{5}{12} + \frac{4}{12} + \frac{3}{12} \\
 & = \$2
 \end{aligned}$$



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| ⋮ | ⋮ | ⋮ | ⋮ |
| N | $\frac{N-1}{N}$ | N | $\frac{N-1}{2N}$ |
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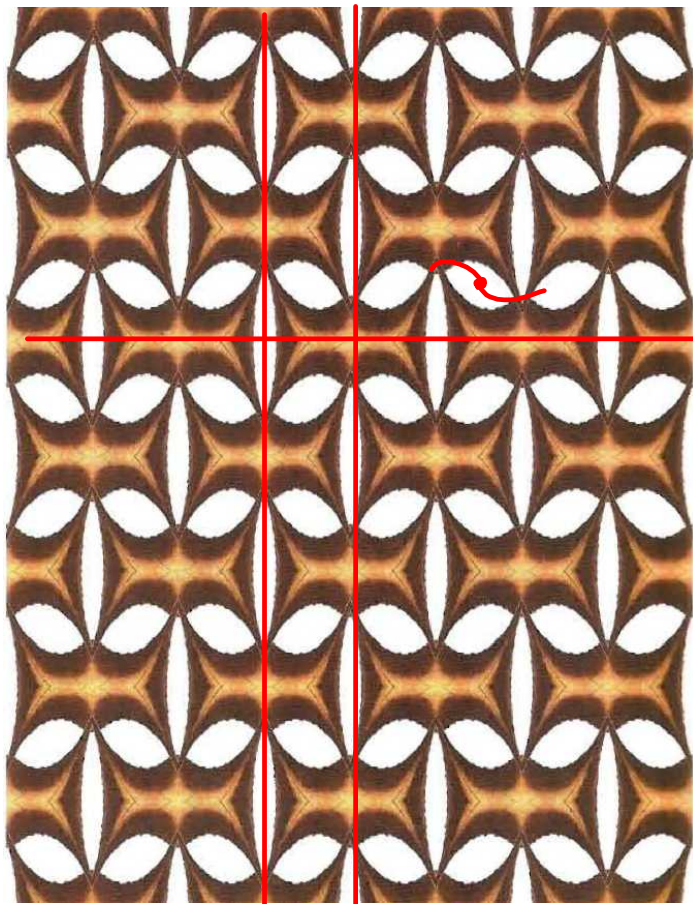
$$\begin{aligned}
 & * 4 4 2 \\
 & \$ 1 + \frac{3}{8} + \frac{3}{8} + \frac{2}{8} \\
 & = \$ 2
 \end{aligned}$$



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Table 3.1. Costs of symbols in signatures.

$$\begin{aligned}
 & 3 * 3 \\
 & \$ \frac{2}{3} + \$ 1 + \$ \frac{1}{3} \\
 & = \$ 2
 \end{aligned}$$



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|--------|-----------------|--------|------------------|
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| 3 | $\frac{2}{3}$ | 3 | $\frac{1}{3}$ |
| 4 | $\frac{3}{4}$ | 4 | $\frac{1}{6}$ |
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| ⋮ | ⋮ | ⋮ | ⋮ |
| N | $\frac{N-1}{N}$ | N | $\frac{N-1}{2N}$ |
| ∞ | 1 | ∞ | $\frac{1}{2}$ |

Table 3.1. Costs of symbols in signatures.

$$2 * 2 \quad 2$$

$$\frac{1}{2} + 1 + \frac{1}{4} + \frac{1}{4}$$