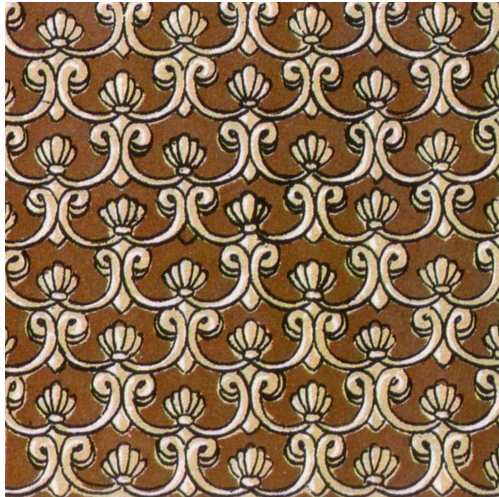


Planar Symmetry & Wallpaper Groups



There are exactly 17 so-called "wallpaper groups," which, in orbifold notation, can be classified as:

Planar Symmetry & Wallpaper Groups

1. o
2. xx
3. *x
4. **
5. 632
6. *632
7. 333
8. *333
9. 3*3
10. 442
11. *442
12. 4*2
13. 2222
14. 22x
15. 22*
16. *2222
17. 2*22

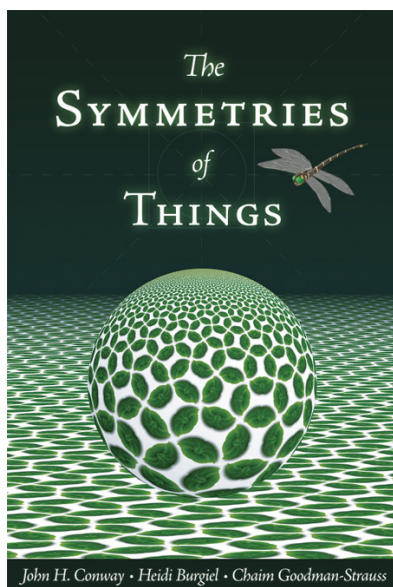
where the notations mean the following:

o is a "wonder ring" or "wandering" and indicates two independent directions of **translation**

x is a "miracle" or **mirrorless glide reflection**

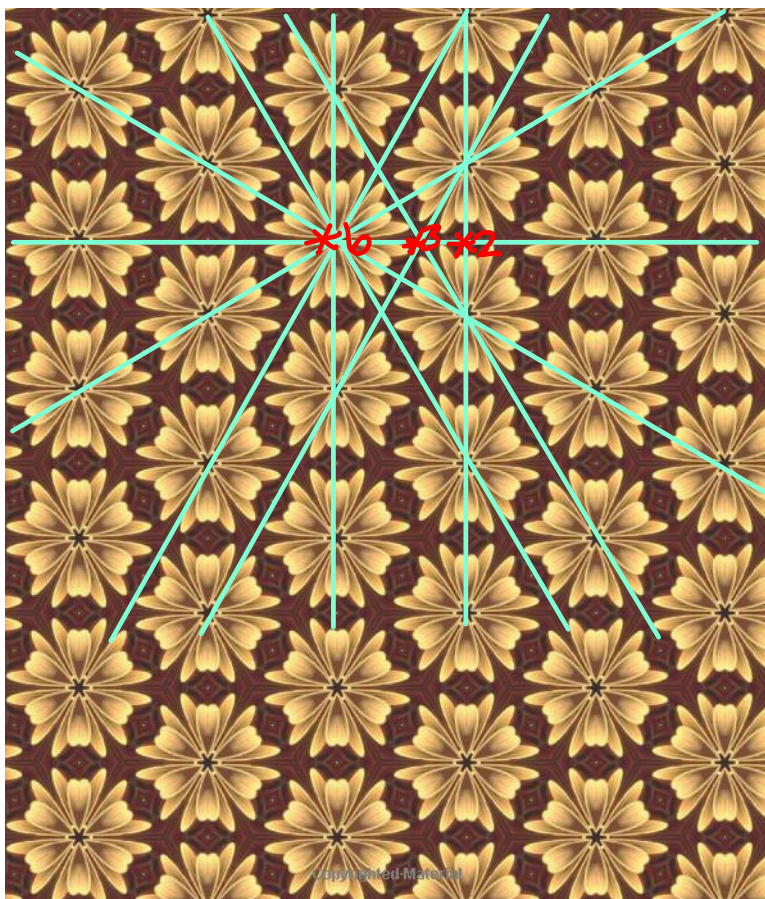
***** is a **mirror line**, and any numbers after a * indicate different points with that many mirror lines passing through those points

numbers before a * or numbers without any other notation indicate points with that number-fold **rotational symmetry**



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{2}{3}$	3	$\frac{1}{3}$
4	$\frac{3}{4}$	4	$\frac{3}{8}$
5	$\frac{4}{5}$	5	$\frac{2}{5}$
6	$\frac{5}{6}$	6	$\frac{5}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

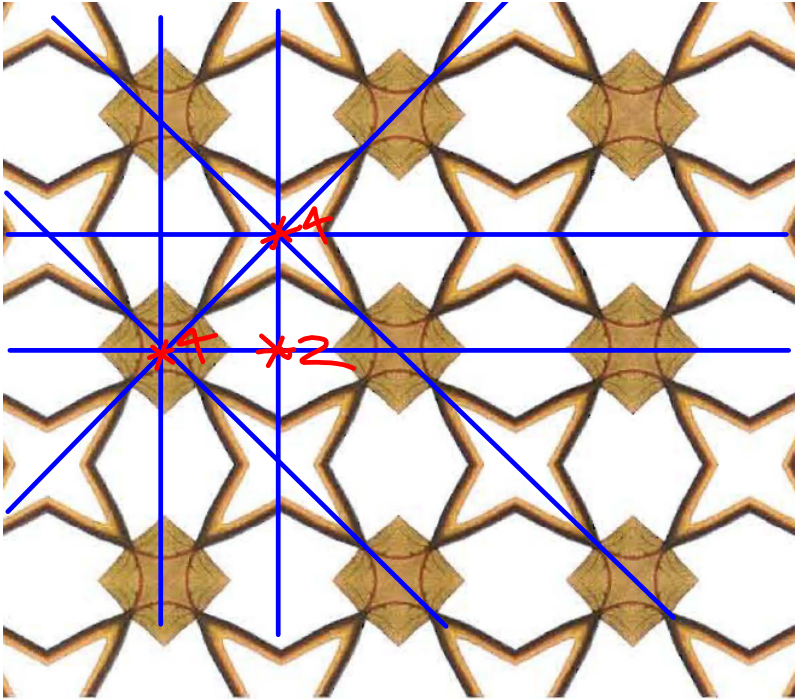
Table 3.1. Costs of symbols in signatures.



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{2}{3}$	3	$\frac{1}{3}$
4	$\frac{3}{4}$	4	$\frac{3}{8}$
5	$\frac{4}{5}$	5	$\frac{2}{5}$
6	$\frac{5}{6}$	6	$\frac{5}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.1. Costs of symbols in signatures.

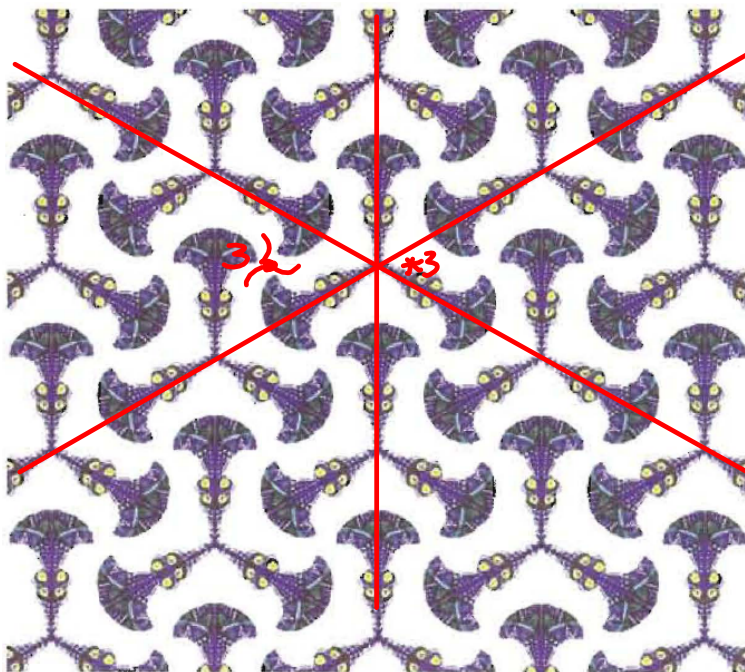
* 6 3 2
 $\$1 + \frac{5}{12} + \frac{1}{3} + \frac{1}{4}$
 $\frac{12}{12} + \frac{5}{12} + \frac{4}{12} + \frac{3}{12} = \2



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{2}{3}$	3	$\frac{1}{3}$
4	$\frac{3}{4}$	4	$\frac{3}{8}$
5	$\frac{4}{5}$	5	$\frac{2}{5}$
6	$\frac{5}{6}$	6	$\frac{5}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.1. Costs of symbols in signatures.

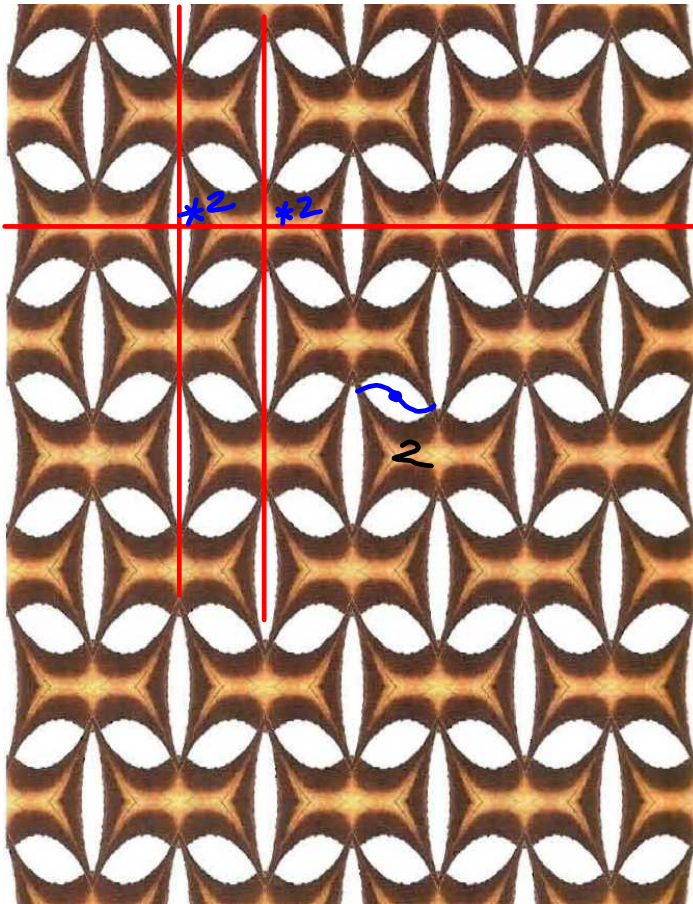
$$*4 + 2 + 1 + \frac{3}{8} + \frac{3}{8} + \frac{1}{4} = \$2$$



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{2}{3}$	3	$\frac{1}{3}$
4	$\frac{3}{4}$	4	$\frac{3}{8}$
5	$\frac{4}{5}$	5	$\frac{2}{5}$
6	$\frac{5}{6}$	6	$\frac{5}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.1. Costs of symbols in signatures.

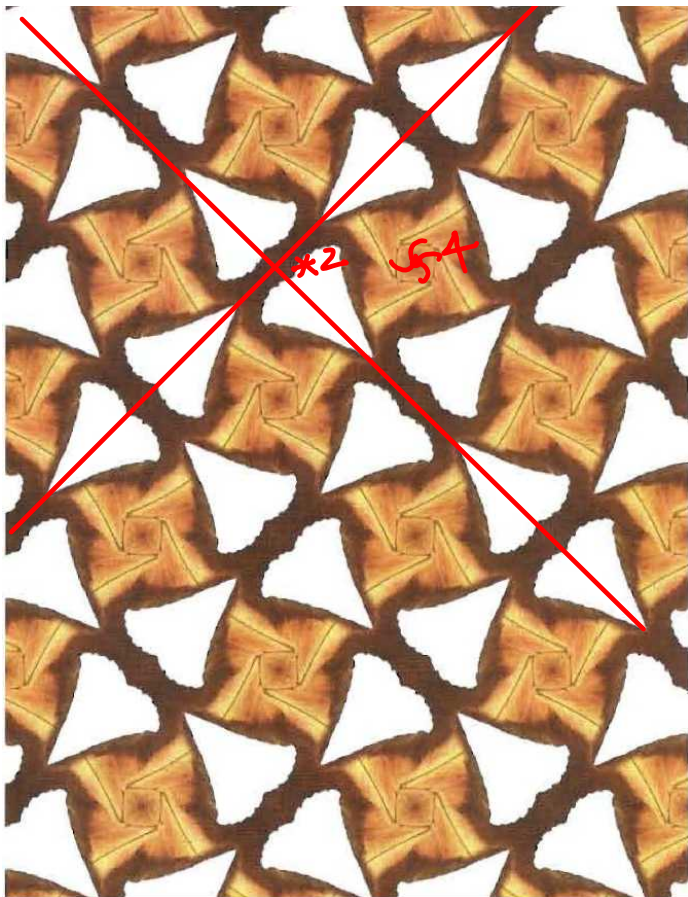
$$3 \times 3 + \frac{2}{3} + 1 + \frac{1}{3} = \$2$$



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{1}{3}$	3	$\frac{1}{3}$
4	$\frac{1}{4}$	4	$\frac{1}{6}$
5	$\frac{1}{5}$	5	$\frac{1}{10}$
6	$\frac{1}{6}$	6	$\frac{1}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.I. Costs of symbols in signatures.

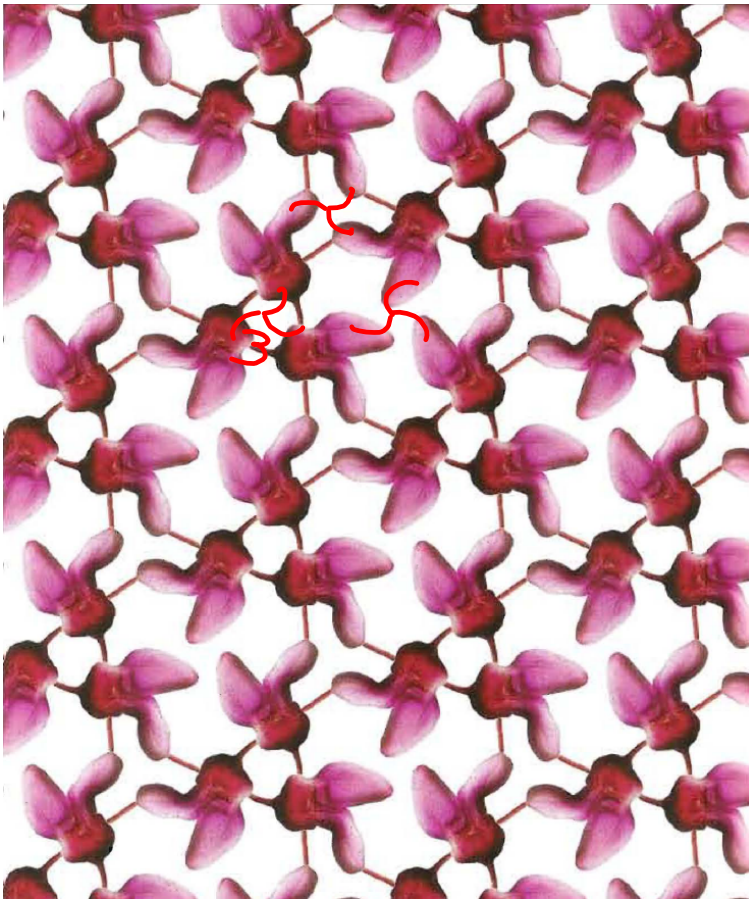
$2 * 22$
 $\frac{1}{2} + 1 + \frac{1}{4} + \frac{1}{4} = \boxed{\$2}$



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{1}{3}$	3	$\frac{1}{3}$
4	$\frac{1}{4}$	4	$\frac{1}{6}$
5	$\frac{1}{5}$	5	$\frac{1}{10}$
6	$\frac{1}{6}$	6	$\frac{1}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.I. Costs of symbols in signatures.

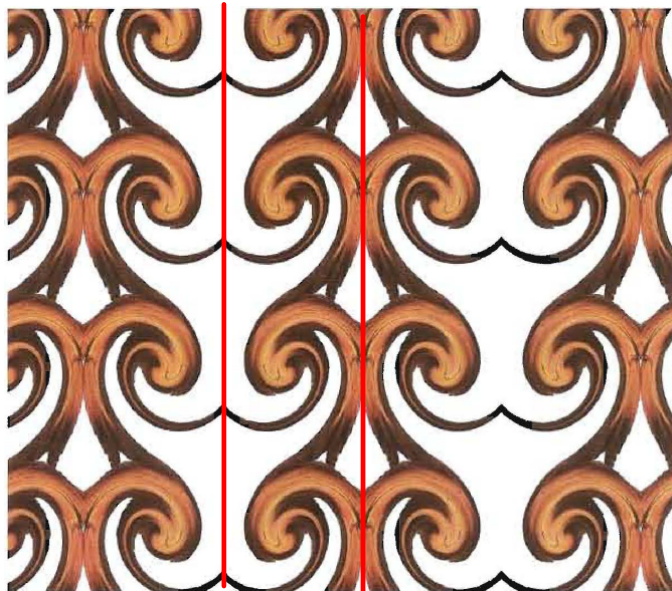
$4 * 2$
 $\frac{3}{4} + 1 + \frac{1}{4} = \boxed{\$2}$



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{1}{3}$	3	$\frac{1}{6}$
4	$\frac{1}{4}$	4	$\frac{1}{8}$
5	$\frac{1}{5}$	5	$\frac{1}{10}$
6	$\frac{1}{6}$	6	$\frac{1}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.I. Costs of symbols in signatures.

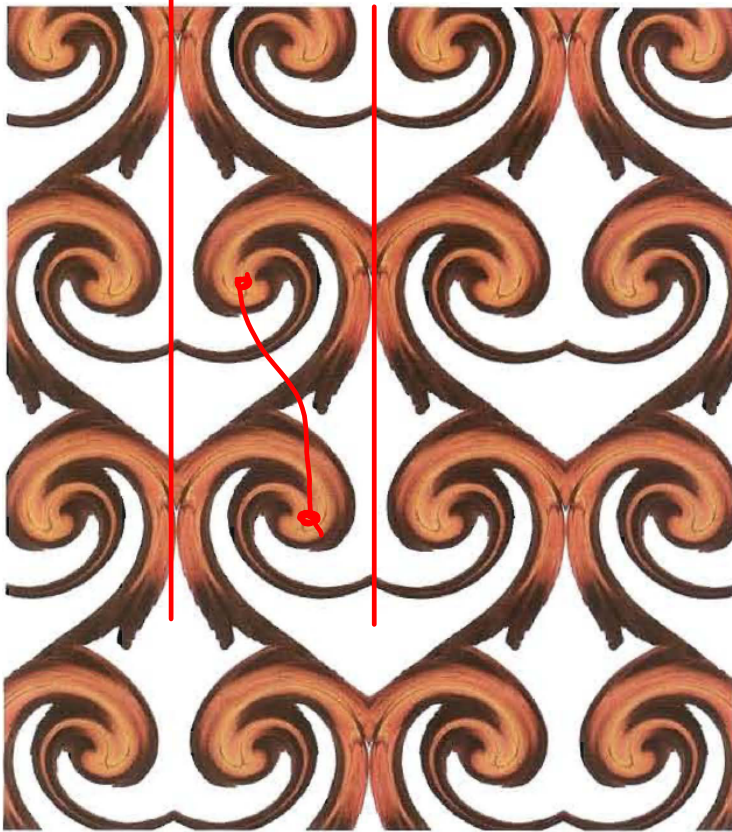
$$333 + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \$2$$



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{1}{3}$	3	$\frac{1}{6}$
4	$\frac{1}{4}$	4	$\frac{1}{8}$
5	$\frac{1}{5}$	5	$\frac{1}{10}$
6	$\frac{1}{6}$	6	$\frac{1}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.I. Costs of symbols in signatures.

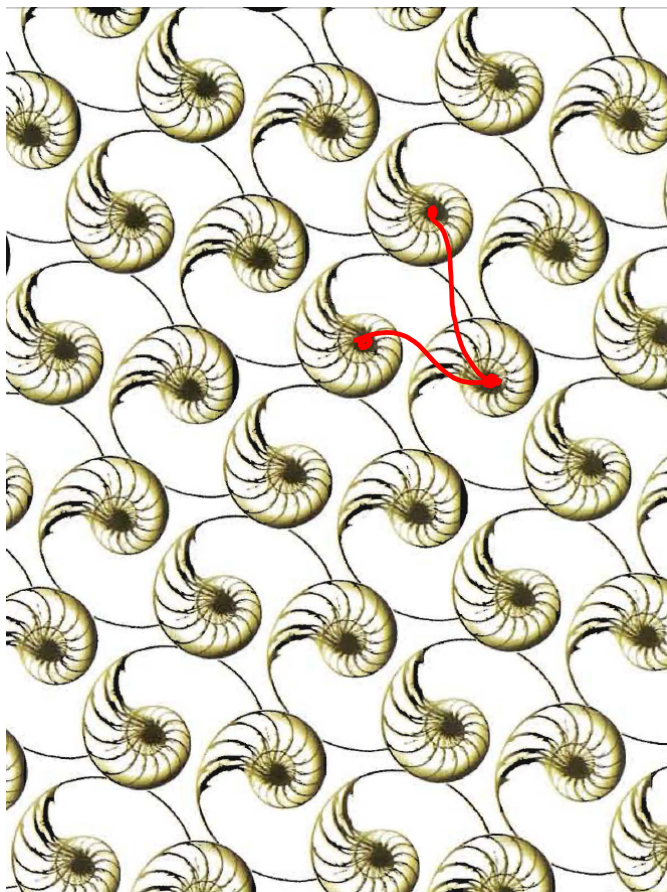
$$* * + 1 + 1 = \$2$$



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{1}{3}$	3	$\frac{1}{3}$
4	$\frac{1}{4}$	4	$\frac{1}{8}$
5	$\frac{1}{5}$	5	$\frac{1}{5}$
6	$\frac{1}{6}$	6	$\frac{1}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.1. Costs of symbols in signatures.

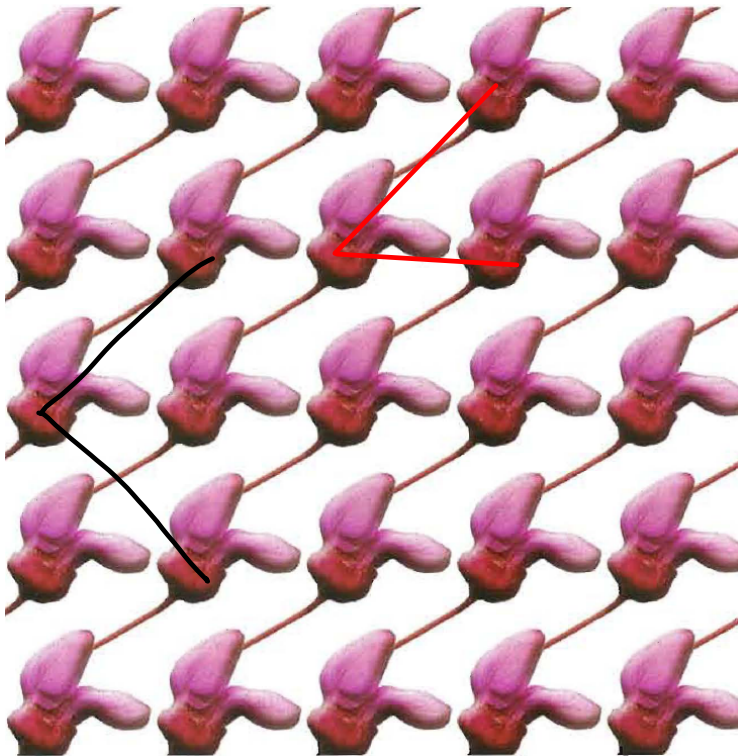
$X* \text{ or } *X$
 $1+1 = \$2$



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{1}{3}$	3	$\frac{1}{3}$
4	$\frac{1}{4}$	4	$\frac{1}{8}$
5	$\frac{1}{5}$	5	$\frac{1}{5}$
6	$\frac{1}{6}$	6	$\frac{1}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.1. Costs of symbols in signatures.

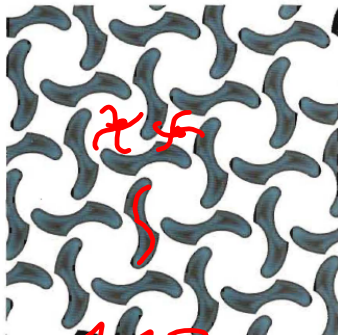
XX
 $1+1 = \$2$



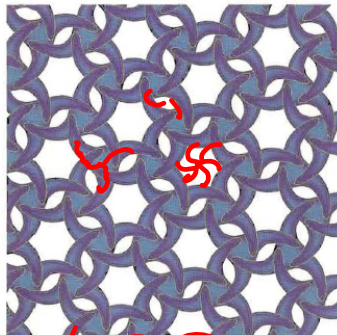
Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{1}{3}$	3	$\frac{1}{3}$
4	$\frac{1}{4}$	4	$\frac{1}{8}$
5	$\frac{1}{5}$	5	$\frac{1}{5}$
6	$\frac{1}{6}$	6	$\frac{1}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.I. Costs of symbols in signatures.

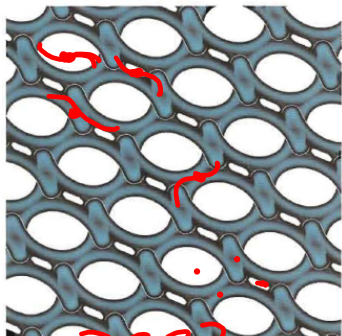
○ = \$2



4 4 2



6 3 2



2 2 2



Symbol	Cost (\$)	Symbol	Cost (\$)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{1}{3}$	3	$\frac{1}{3}$
4	$\frac{1}{4}$	4	$\frac{1}{8}$
5	$\frac{1}{5}$	5	$\frac{1}{5}$
6	$\frac{1}{6}$	6	$\frac{1}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.I. Costs of symbols in signatures.

