Munkres 1.1

Set Theory

A set

a element of a set

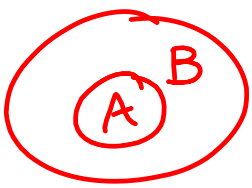
$a \in A$

subsets

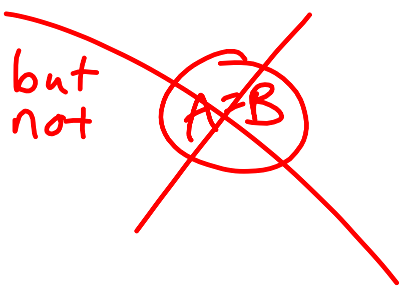
$A \subset B$

$A \subsetneq B$

proper subset



but not



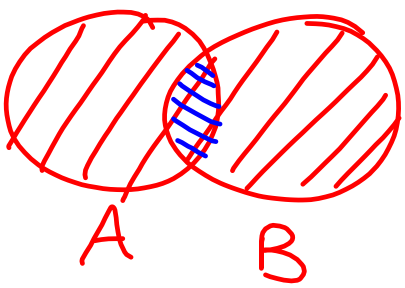
Subset	proper subset
$A \subseteq B$	$A \subset B$

$A \subset B \qquad B \supset A$

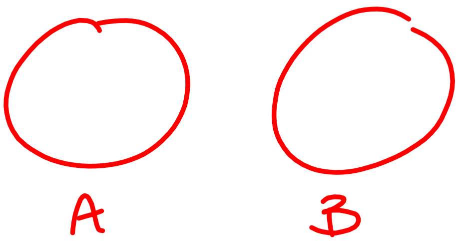
$x < y \qquad y > x$

union & intersection

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$



$A \cap B = \{x \mid x \in A \ \underline{\text{and}} \ x \in B\}$



$A \cap B = ? = \emptyset$

$\emptyset$ = "empty set" =

= set containing no elements

$A \subset B$

If $x \in A$, then $x \in B$.

$\emptyset \subset A$ "vacuously true" for any set A

$A \cap \emptyset = \emptyset$

$A \cup \emptyset = A$

$A = B$

$\Rightarrow A \subseteq B$ and $B \subseteq A$

Case 1 $A \subseteq B$

Assume $x \in A$. To show $x \in B$

. . .

Case 2. $B \subseteq A$

Assume $y \in B$. To show $y \in A$.

Formal Logic

If A, then B.

(If A is true, then B is true.) If it is raining, then the streets are wet.

$A \Rightarrow B$ "A implies B"

contrapositive: If (not B) then (not A).

$\sim B \Rightarrow \sim A$

If the streets are not wet, then it is not raining.

Logically equivalent to original statement

converse: If B, then A. $B \Rightarrow A$

If the streets are wet, then it is raining.

NOT logically equivalent to original statement

$A = B$

$A \Rightarrow B$ and $B \Rightarrow A$

A if and only if B

<u>Thm</u>: For every $x \in A$, statement P holds.

$\forall$ $x \in A$, statement P holds.
"for all"

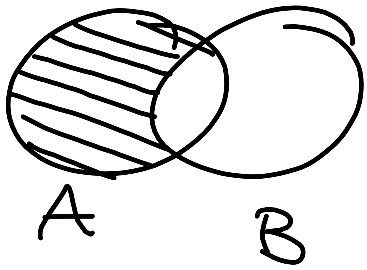
<u>Contrapositive</u>:

If statement P does not hold, then
not every x is an element of A

$\exists$ $x \in A$ such that statement P does not hold.
"there exists"

Difference of Sets

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$



Set theory rules

"Distributive Properties"

* $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De Morgan's Laws:

$$A - (B \cup C) = (A - B) \cap (A - C)$$
$$A - (B \cap C) = (A - B) \cup (A - C)$$

Theorem: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof:

$\subseteq$: Let $x \in A \cap (B \cup C)$.

We to show that $x \in (A \cap B) \cup (A \cap C)$.

$x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in B \cup C$.

$x \in B \cup C \Rightarrow x \in B$ or $x \in C$.

Case 1: $x \in A$ and $x \in B$

$\Rightarrow x \in A \cap B$.

$A \cap B \subseteq (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$.

Case 2: $x \in A$ and $x \in C$.

$\Rightarrow x \in A \cap C \subseteq (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$.

$\supseteq$: Let $x \in (A \cap B) \cup (A \cap C)$

To show $x \in A \cap (B \cup C)$.