Set theory rules

"Distributive Properties"

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

De Morgan's Laws:

$$A - (B \cup C) = (A - B) \cap (A - C)$$
$$A - (B \cap C) = (A - B) \cup (A - C)$$

Theorem: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof:

$\subseteq$: Let $x \in A \cap (B \cup C)$.

We to show that $x \in (A \cap B) \cup (A \cap C)$.

$x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in B \cup C$.

$x \in B \cup C \Rightarrow x \in B$ or $x \in C$.

Case 1: $x \in A$ and $x \in B$

$\Rightarrow x \in A \cap B$.

$A \cap B \subseteq (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$.

Case 2: $x \in A$ and $x \in C$.

$\Rightarrow x \in A \cap C \subseteq (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$.

$\supseteq$: Let $x \in (A \cap B) \cup (A \cap C)$

To show $x \in A \cap (B \cup C)$.

<u>Theorem</u>: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\supseteq$: Let $y \in (A \cap B) \cup (A \cap C)$
To show $y \in A \cap (B \cup C)$.

$y \in (A \cap B) \cup (A \cap C)$
$\Rightarrow y \in A \cap B$ or $y \in A \cap C$

Case 1: $y \in A \cap B \Rightarrow y \in A$ and $y \in B$
$B \subseteq B \cup C \Rightarrow A \cap B \subseteq B \subseteq B \cup C$
$\Rightarrow y \in B \cup C$.
$y \in B \cup C$ and $y \in A \Rightarrow y \in A \cap (B \cup C)$

Case 2: $y \in A \cap C \Rightarrow y \in A$ and $y \in C$
$C \subseteq B \cup C \Rightarrow A \cap C \subseteq C \subseteq B \cup C$
$\Rightarrow y \in B \cup C$
$y \in A$ and $y \in B \cup C \Rightarrow y \in A \cap (B \cup C)$. $\square$

$$A - (B \cup C) = (A - B) \cap (A - C)$$

$A = B$ if $A \subseteq B$ and $B \subseteq A$

$x \in A \cup B$ if $x \in A$ or $x \in B$

$x \in A \cap B$ if $x \in A$ and $x \in B$

$x \in A - B$ if $x \in A$ but $x \notin B$

"is an element of"

$A \subseteq B \Rightarrow$ if $x \in A$, then $x \in B$.

$$A-(B\cup C)=(A-B)\cap(A-C)$$

Proof :

$\subseteq$: Let $x\in A-(B\cup C)$.

(To show $x\in(A-B)\cap(A-C)$.)

$x\in A-(B\cup C) \Rightarrow x\in A$ and $x\notin(B\cup C)$.

$x\notin(B\cup C) \Rightarrow x\notin B$ and $x\notin C$.

$x\in A$ and $x\notin B \Rightarrow x\in A-B$

$x\in A$ and $x\notin C \Rightarrow x\in A-C$

$x\in A-B$ and $x\in A-C \Rightarrow$

$x\in(A-B)\cap(A-C)$.

$$A-(B\cup C)=(A-B)\cap(A-C)$$

proof, cont.

$\supseteq$: Let $y\in(A-B)\cap(A-C)$.

We want to show that $y\in A-(B\cup C)$.

$y\in(A-B)\cap(A-C) \Rightarrow y\in A-B$ and $y\in A-C$.

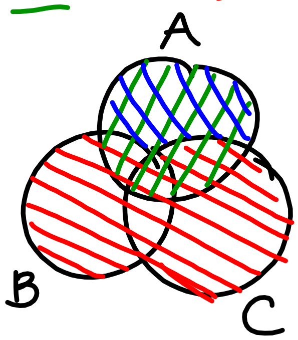
$y\in A-B \Rightarrow y\in A$ and $y\notin B$.

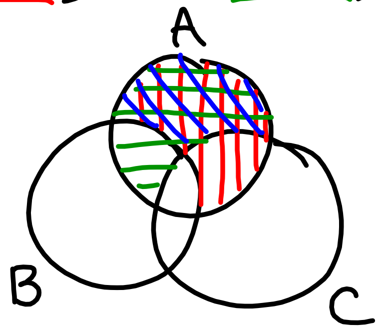
$y\in A-C \Rightarrow y\in A$ and $y\notin C$.

$y\notin B$ and $y\notin C \Rightarrow y\notin B\cup C$.

$y\in A$ and $y\notin B\cup C \Rightarrow y\in A-(B\cup C)$.

$$A-(B\cup C)=(A-B)\cap(A-C)$$





A - capital letters for sets

a - lowercase letters for elements of set

$\mathcal{A}$ - script capitals for collections of sets

"Power Set" of set A : $\mathcal{P}(A)$

is the set of all of the subsets of A.

$$A = \{a, b, c, d\}$$

$$\mathcal{P}(A) = \{ \emptyset, A, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\} \}$$

If A contains n elements, $\mathcal{P}(A)$ will contain 2^n elements.

$$\bigcup_{A \in \mathcal{A}} A = \{x \mid x \in A \text{ for some } A \in \mathcal{A}\}$$
(at least one)

$$\bigcap_{A \in \mathcal{A}} A = \{x \mid x \in A \text{ for all } A \in \mathcal{A}\}$$
$x \in A \ \forall \ A \in \mathcal{A}$

$\bigcap_{A \in \mathcal{A}} A \subseteq A$ for all A

$A \subseteq \bigcup_{A \in \mathcal{A}} A$ for all A

Regular De Morgan

$$A-(B \cap C) = (A-B) \cup (A-C)$$

De Morgan's Law for arbitrary intersections

$$A - \bigcap_{B \in \mathcal{B}} B = \bigcup_{B \in \mathcal{B}} (A-B)$$

Proof :

$\subseteq$: Let $x \in A - \bigcap_{B \in \mathcal{B}} B \Rightarrow x \in A$ and $x \notin \bigcap_{B \in \mathcal{B}} B$.

$x \notin \bigcap_{B \in \mathcal{B}} B \Rightarrow x \notin B_1$ for at least one $B_1 \in \mathcal{B}$.

$x \in A$ and $x \notin B_1$ for some $B_1 \Rightarrow$
$x \in A - B_1$ for at least one B_1

$A-B_1 \subseteq \bigcup_{B \in \mathcal{B}} A-B_i$ for all $B \in \mathcal{B} \Rightarrow x \in \bigcup_{B \in \mathcal{B}} A-B$.

$(A-B_1) \cup (A-B_2) \cup (A-B_3) \cup \cdots$

Hw #1:

1.1 #1-10 due next Wed.