§1.1

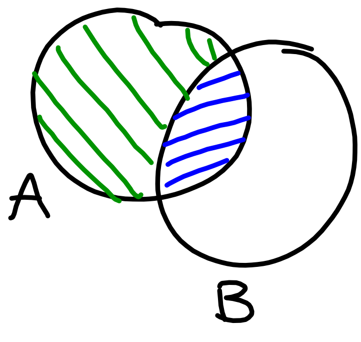
2a. $\underbrace{A \subset B \text{ and } A \subset C}_{\Rightarrow A \subset B \cap C} \iff \underbrace{A \subset (B \cup C)}_{\Rightarrow A \subset B \text{ or } A \subset C}$

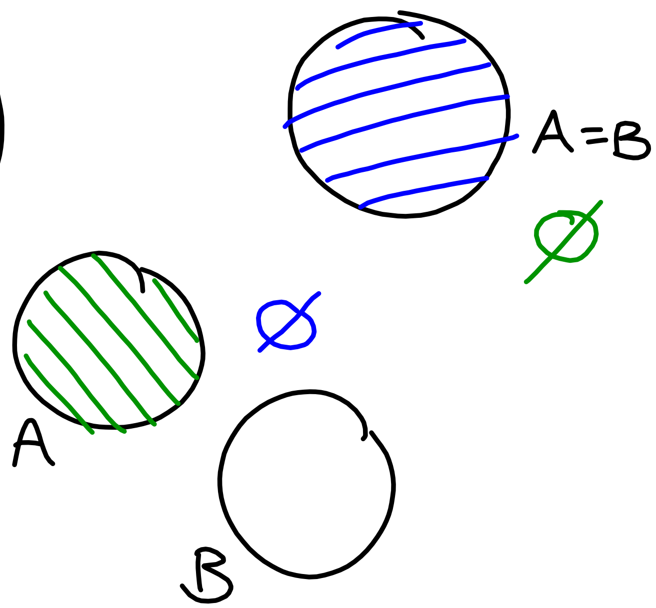
$\iff$ false

$\implies$ true

$\impliedby$ false

2i. $(A \cap B) \cup (A - B) = A$ True





Cartesian Product

$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$

$\mathbb{R} \times \mathbb{R}$ = 2-dim'l cartesian plane of (x,y) coordinates

2m. $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$

(a,b) $a \in A$ $b \in B$ $\cup$ (c,d) $c \in C$ $d \in D$

$\{x \mid x \in A \text{ or } x \in C\} \times \{y \mid y \in B \text{ or } y \in D\}$

(x,y)

$x \in A$ or C

$y \in B$ or D

$=$ false

$\subseteq$ true

$\supseteq$ false contains (a,d) & (b,c)

If A, then B.

If $x > 0$, then $x^3 \neq 0$. true

Contrapositive (If $\sim B$, then $\sim A$) "not"

If $x^3 = 0$, then $x \leq 0$. true

Converse (If B, then A)

If $x^3 \neq 0$, then $x > 0$. false

If $x^3 = -8$, then $x = -2$.

If A, then B.

B $\lim_{x \to a} f(x) = L$ if for every ($\forall$) $\varepsilon > 0$,
there exists ($\exists$) $\delta > 0$ such that
$|x-a| < \delta$ implies $|f(x) - L| < \varepsilon$. } A

(ε-δ definition of the limit)

Contrapositive: If ~B, then ~A.

If $\lim_{x \to a} f(x) \neq L$, then there exists ($\exists$) $\varepsilon > 0$,
such that for any ($\forall$) $\delta > 0$, if
$|x-a| < \delta$, we have $|f(x) - L| \geq \varepsilon$.

§2 Functions

rule of assignment is a subset r of the cartesian product $C \times D$ of two sets, having the property that each element of C appears as the first coordinate of at most one ordered pair belonging to r.

(how we defined "function" in Precal)

$$(c_1, d_1) = (c_1, d_2) \Rightarrow d_1 = d_2$$

The domain of a rule of assignment $r \subset C \times D$ is the subset of C consisting of all first coordinates of r.

The image set of r is the subset of D consisting of all the second coordinates.

domain of $r = \{c \in C \mid \exists\, d \in D \text{ s.t. } (c,d) \in r\}$

image of $r = \{d \in D \mid \exists\, c \in D \text{ s.t. } (c,d) \in r\}$

A function f is a rule of assignment r, togther with a set B that contains the image set of r.

the domain of f is the domain of r

the image of f is the image of r

the range of f is the set B.

$f: A \longrightarrow B$ (f is a function with domain A and range B)

$(f: \mathbb{R} \longrightarrow \mathbb{R})$

$f(x) = x^2$

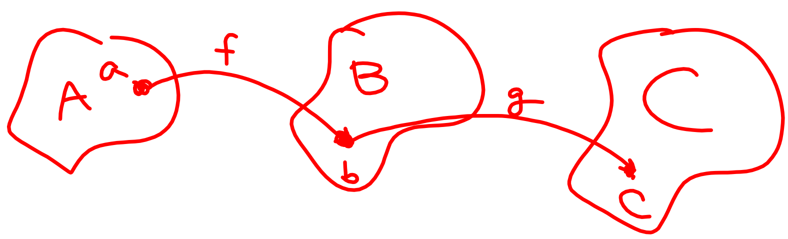
If $f: A \to B$, and $A_0 \subset A$,
the <u>restriction</u> of f to A_0 as
$\{(a, f(a)) \mid a \in A_0\}$

$f(x) = \sin x$ $\quad f: \mathbb{R} \to \mathbb{R}$

to construct $f^{-1}(x) = \sin^{-1}(x)$,
we first restrict $f: [-\frac{\pi}{2}, \frac{\pi}{2}] \to \mathbb{R}$

Given $f: A \to B$ and $g: B \to C$,
we can define the <u>composition</u>
$f \circ g: A \to C$ as
$\{(a,c) \mid \text{For some } b \in B,\ f(a)=b \text{ and } g(b)=c.\}$

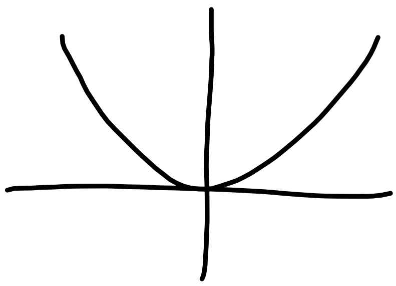
Note that $f \circ g$ is only defined when the range of f equals the domain of g.



$f: A \to B$ is <u>injective</u> (or <u>one-to-one</u>)
if $f(a_1) = f(a_2)$, then $a_1 = a_2$.

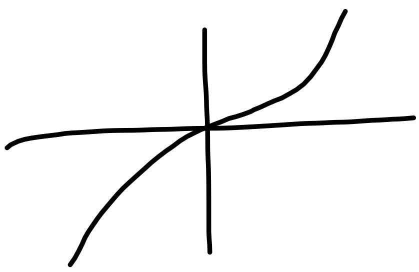
$f: A \to B$ is <u>surjective</u> (or onto)
if for every $b \in B$, there exists $a \in A$ such that $f(a) = b$.

$f: \mathbb{R} \to \mathbb{R}$
$f(x) = x^2$ is <u>not</u> onto



range is not all of $\mathbb{R}$

$f(x) = x^3$ <u>is</u> onto



If f is both one-to-one and onto, we call it <u>bijective</u> and we can define the inverse of f.