§1.2 - Functions

A <u>function</u> f is a rule of assignment (each input maps to one output) together with a set B (range) that contains the image set of the rule.

f is <u>one-to-one</u> or <u>injective</u> if
$f(a)=f(b)$ implies that $a=b$
for all $a,b \in$ domain of f

$f:A\to B$ is <u>onto</u> or <u>surjective</u> if
for every $b\in B$, there exists $a\in A$
such that $f(a)=b$.

If $f:A\to B$ is both injective & surjective, it is called <u>bijective</u> and has a unique inverse function $f^{-1}:B\to A$

(For a bijective function $f:A\to B$)

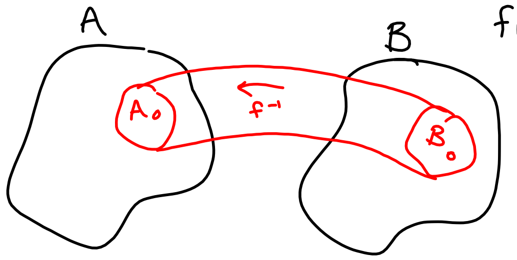
If $x\in f^{-1}(B) \Rightarrow f(x)\in B$

If $y\in f(A) \Rightarrow f^{-1}(y)\in A$

1. (b) Let $f: A \to B$.
$A_0 \subset A$
$B_0 \subset B$
Show that $f(f^{-1}(B_0)) \subset B_0$
and that equality holds if f is onto.

Recall: To show that $A \subset B$, take an arbitrary $x \in A$ and work toward $x \in B$.

Proof: Let $x \in f(f^{-1}(B_0))$.
$f^{-1}(B_0) = A_0 \subset A \Rightarrow x \in f(A_0) \Rightarrow x \in f(A)$
Since $f: A \to B$, $\Rightarrow x \in B$) $\boxed{f^{-1}(x) \in A_0} \Rightarrow$
$f(f^{-1}(x)) \in B_0$
$\Rightarrow x \in B_0$



the preimage of B_0 under f is (p 19)
$A_0 = f^{-1}(B_0) = \{a \mid f(a) \in B_0\}$

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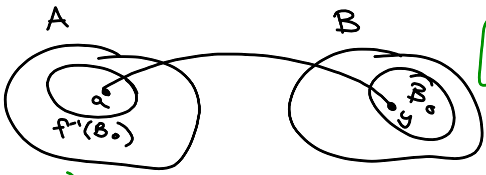
Show that $f(f^{-1}(B_0)) \subset B_0$
and that *equality holds if f is onto.

To show that $B_0 \subset f(f^{-1}(B_0))$.

Let $y \in B_0$. Since f is onto, $\exists\, a \in f^{-1}(B_0)$ such that $f(a) = y$. equivalently, $a = f^{-1}(y)$
$a \in f^{-1}(B_0) \Rightarrow f(a) \in B_0$, i.e. $f(a) = b$ for some $b \in B_0$. $f(a) = y \Rightarrow y = b$



$\to y = f(f^{-1}(y))$

$f(f^{-1}(y)) \in f(f^{-1}(B_0))$; i.e. $y \in f(f^{-1}(B_0))$
i.e. $B_0 \subset f(f^{-1}(B_0))$. $\square$