Part I: Write the *negation* of each statement.

1. Statement: There exists an element $x \in \mathbb{Z}$ such that $x^2 = 4$.

   Negation: For all elements $x \in \mathbb{Z}$, it is true that $x^2 \neq 4$.

   $\forall x \in \mathbb{Z}$, it is not true that $x^2 = 4$.

2. Statement: For all elements $x \in \mathbb{R}$, it is true that $|x| \geq 0$.

   Negation: There exists $x \in \mathbb{R}$ such that $|x| < 0$.

Part II: Write the *contrapositive* and *converse* for each statement, and determine which (if any) of the statements is true. No proofs are necessary, though proving or finding a counter-example will help if you're not sure whether a statement is true or not.

3. Statement: *If* $|x| = 2$, *then* $x = 2$. True / False

   If a, then b.

   $x = -2$

   Contrapositive: True / False

   If not b, then not a.

   If $x \neq 2$, then $|x| \neq 2$

   Converse: True / False

   If b, then a.

   If $x = 2$, then $|x| = 2$

4. Statement: *If* $f(a) = f(b)$, *then* $a = b$. True / False

   <Note: there are no restrictions on f other than that it is a function.>

   $f(x) = x^2$

   $(-2)^2 = 4 = 2^2$, but $-2 \neq 2$

   Contrapositive: True / False

   If $a \neq b$, then $f(a) \neq f(b)$

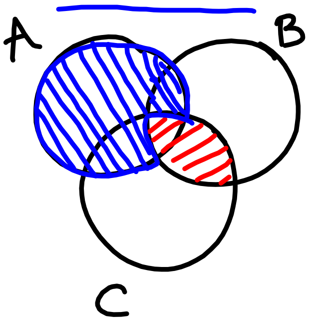
   Converse: True / False

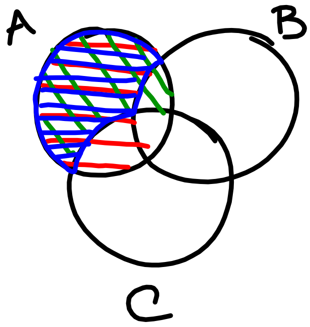
   If $a = b$, then $f(a) = f(b)$

   (true by def. of function)

Part III: Fill in the blank with $=, \subseteq,$ *or* $\supseteq$ to make the statement true for all sets A, B, and C.
If the two sets are equal, prove (as best you can) both subset inclusions.
If the two sets are not equal, prove the inclusion and provide a counter example for the inclusion that fails.

5. $A - (B \cap C) \boxed{\supseteq} (A - B) \cap (A - C)$





Counterexample for $\subseteq$:

$A = \{1,2\}, B = \{1,3\}, C = \{2,3\}$

$2 \in A-(B\cap C) = \{1,2\} - \{3\} = \{1,2\}$

$2 \notin (A-B)\cap(A-C) = \{2\} \cap \{1\} = \emptyset$

Proof: $\supseteq$

Let $x \in (A-B)\cap(A-C)$. $\Rightarrow x \in A-B$ and $x \in A-C$

$x \in A-B \Rightarrow x \in A$ and $x \notin B$

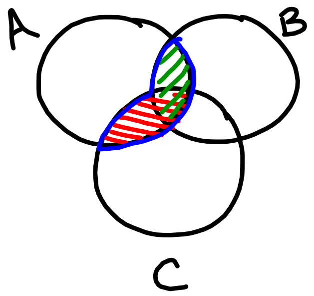
$x \in A-C \Rightarrow x \in A$ and $x \notin C$

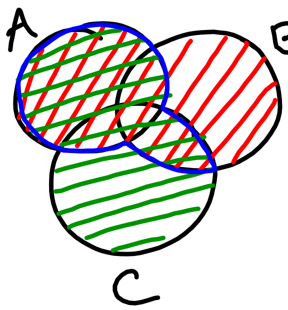
$x \notin B$ and $x \notin C \Rightarrow x \notin B \cap C$

$x \in A$ and $x \notin B\cap C \Longrightarrow x \in A-(B\cap C)$.

Hence $(A-B)\cap(A-C) \subseteq A-(B\cap C)$. $\square$

6. $(A \cap C) \cup (A \cap B) \boxed{\subseteq} (A \cup B) \cap (A \cup C)$





Counterexample for $\supseteq$:

$A = \{1,2\}, B = \{1,3\}, C = \{2,3\}$

$3 \notin (A\cap C)\cup(A\cap B) = \{2\} \cup \{1\} = \{1,2\}$

$3 \in (A\cup B)\cap(A\cup C) = \{1,2,3\} \cap \{1,2,3\}$
$= \{1,2,3\}$

Proof: $\subseteq$

Let $x \in (A\cap C)\cup(A\cap B) \Rightarrow x \in A\cap C$ or $x \in A\cap B$

Case 1: $x \in A \cap C \Rightarrow x \in A$ and $x \in C$

$x \in A \Rightarrow x \in A\cup B$ and $x \in A \cup C$ (since $A \subseteq A\cup B$ and $A \subseteq A \cup C$)

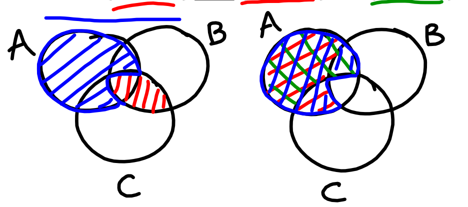
$\Rightarrow x \in (A\cup B)\cap(A\cup C)$.

Case 2: $x \in A\cap B \Rightarrow x \in A$ and $x \in B$

$x \in A \Rightarrow x \in A\cup B$ and $x \in A \cup C \Rightarrow x \in (A\cup B)\cap(A\cup C)$.

Hence $(A\cap C)\cup(A\cap B) \subseteq (A\cup B)\cap(A\cup C)$. $\square$

7. $A - (B \cap C) = (A - B) \cup (A - C)$



Proof: $\subseteq$

Let $x \in A - (B \cap C) \Rightarrow x \in A$ and $x \notin B \cap C$.

$x \notin B \cap C \Rightarrow (x \in B$ and $x \notin C)$ or $(x \notin B$ and $x \in C)$ or $(x \notin B$ and $x \notin C)$

$x \notin B \cap C \Rightarrow x \notin B$ or $x \notin C$.

Case 1: $x \in A$ and $x \notin B \Rightarrow x \in A - B \Rightarrow x \in (A-B) \cup (A-C)$

Case 2: $x \in A$ and $x \notin C \Rightarrow x \in A - C \Rightarrow x \in (A-B) \cup (A-C)$.

Hence $A - (B \cap C) \subseteq (A-B) \cup (A-C)$

$\supseteq$: Let $y \in (A-B) \cup (A-C)$. $\Rightarrow y \in A - B$ or $y \in A - C$.

Case 1: $y \in A - B \Rightarrow y \in A$ and $y \notin B \Rightarrow y \notin B \cap C$

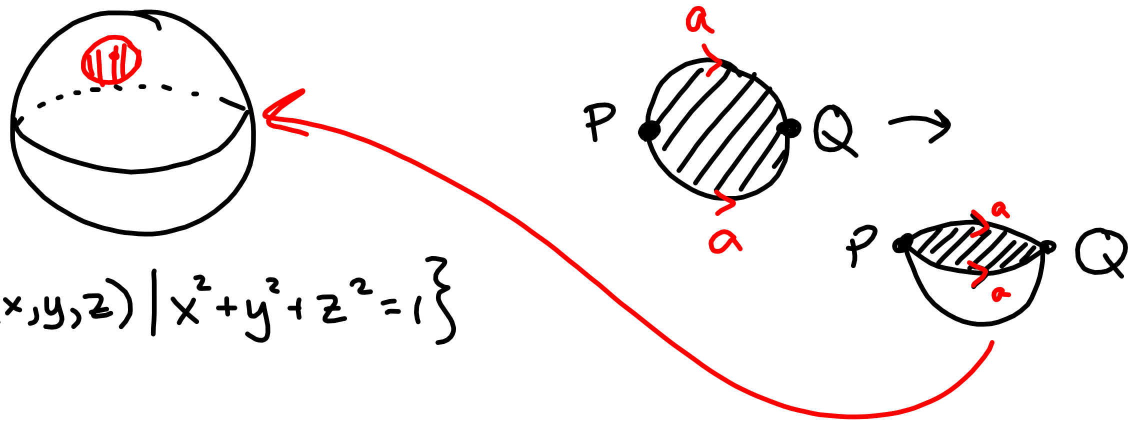
$y \in A$ and $y \notin B \cap C \Rightarrow y \in A - (B \cap C)$.

Case 2: $y \in A - C \Rightarrow y \in A$ and $y \notin C \Rightarrow y \notin B \cap C$

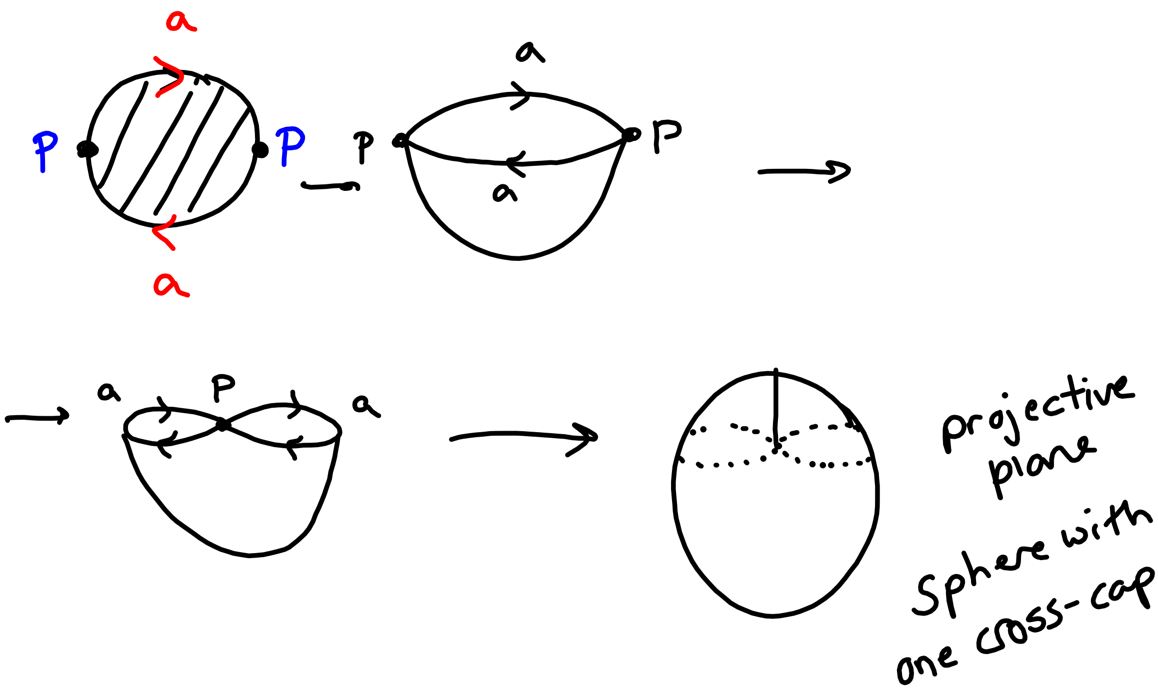
$y \in A$ and $y \notin B \cap C \Rightarrow y \in A - (B \cap C)$.

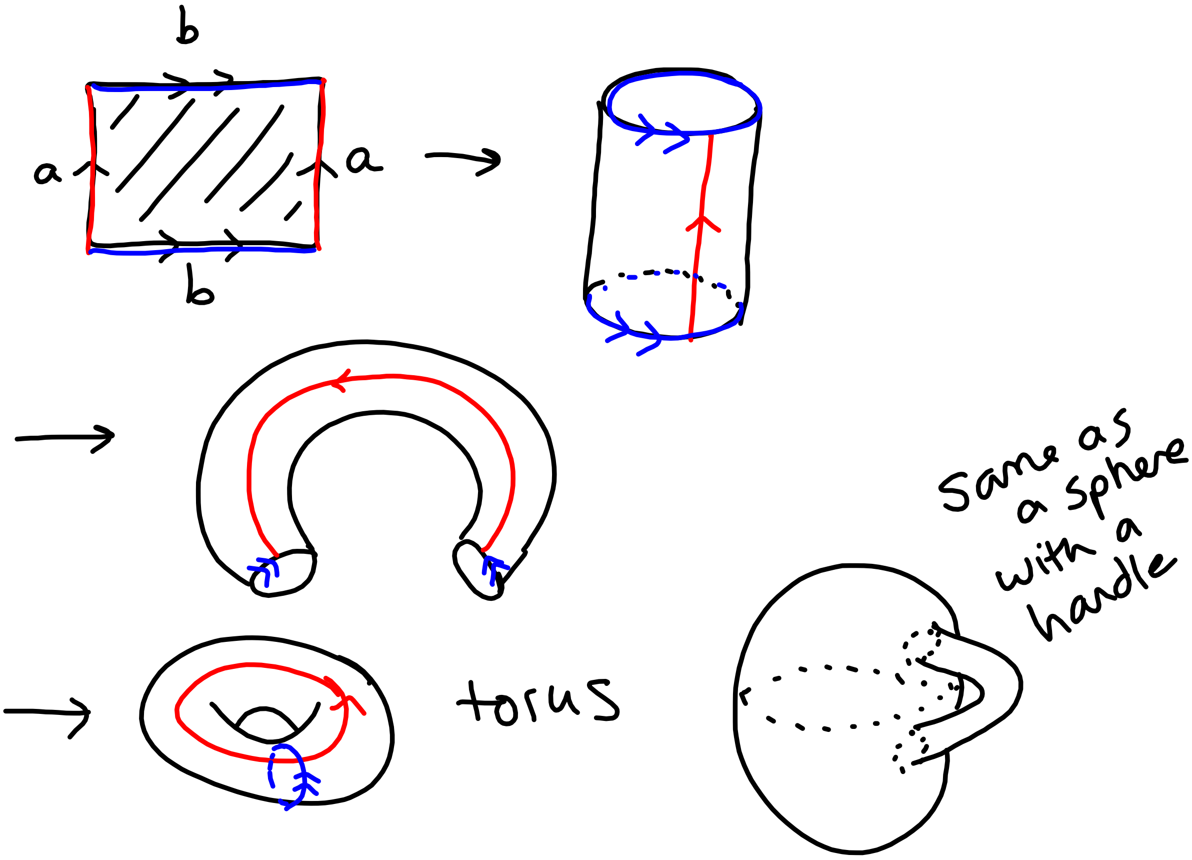
Hence $(A-B) \cup (A-C) \subseteq A - (B \cap C)$. $\square$

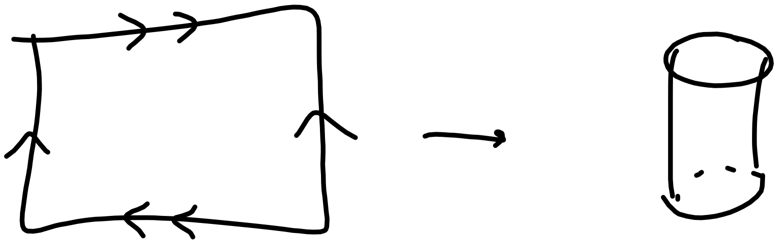
Classification of (2-dimensional) Surfaces

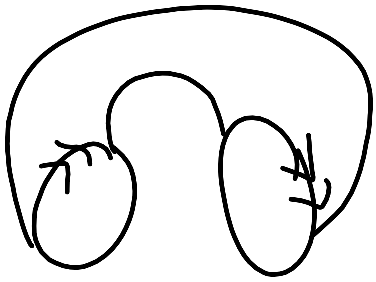


$\{(x,y,z) \mid x^2 + y^2 + z^2 = 1\}$

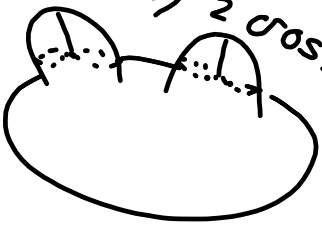

a
P
P
a
a
P
P
a
a
P
a
projective plane
Sphere with one cross-cap


b
a
a
b
torus
Same as a sphere with a handle





same as sphere w/ 2 crosscaps



Klein bottle!