4a. $\Rightarrow\Leftarrow$

b $\Rightarrow$

c. f is injective (prove)
g is not necessarily injective (find counterexample)

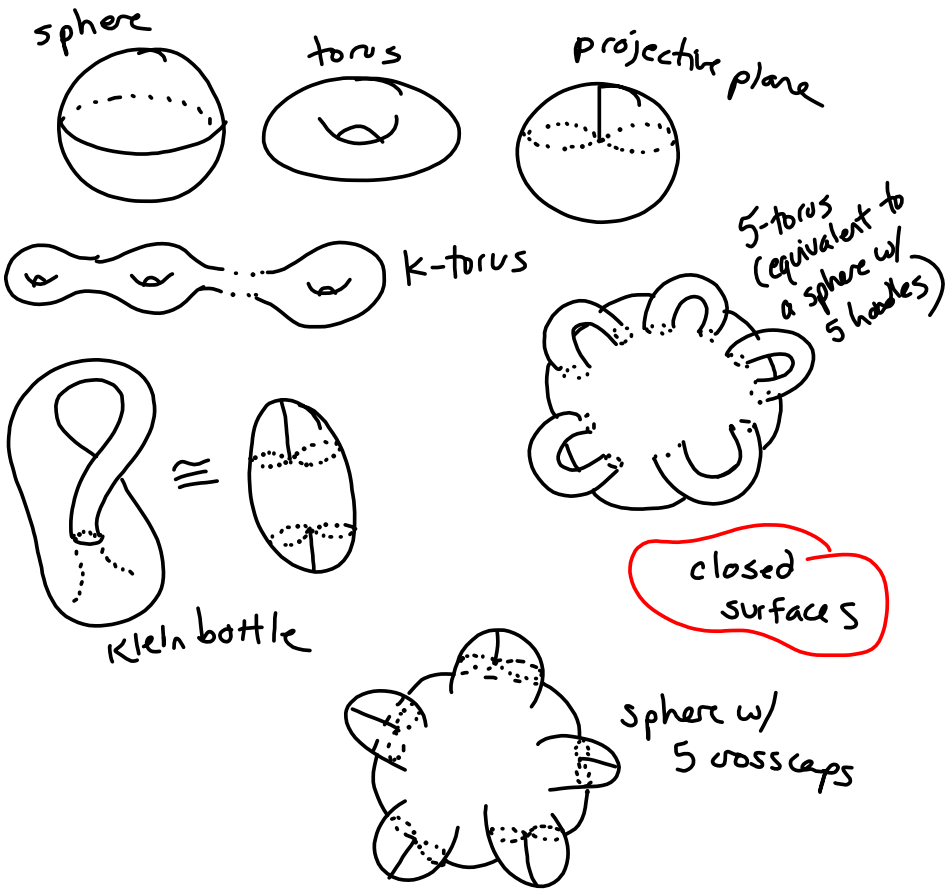
d. $\Rightarrow$

e. g is surjective (prove)
f is not necessarily surjective (find counterexample)

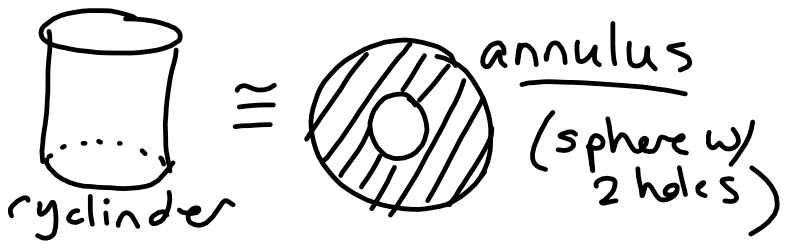
f.
b & d $\Rightarrow$ the composition of bijections is a bijection

c & e $\Rightarrow$ given some bijection $g \circ f$
f must be injective &
g must be surjective

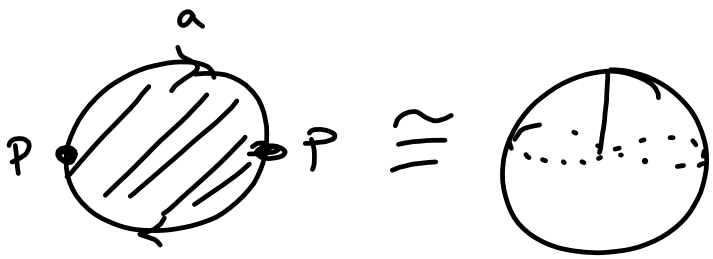
HW #1 corrections by
Wed. 3/26
&
HW #2 corrections by & Quiz/Test
Tues 4/1

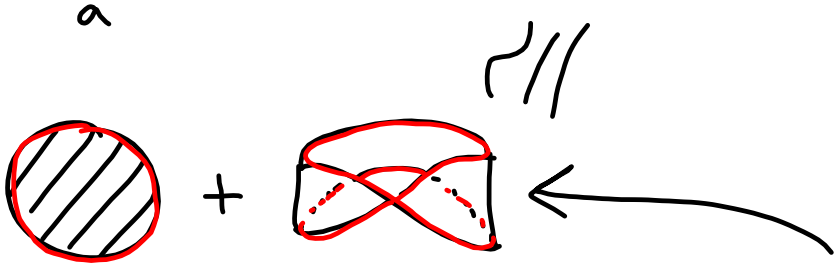

sphere
torus
projective plane
k-torus
5-torus (equivalent to a sphere w/ 5 handles)
Klein bottle
closed surfaces
sphere w/ 5 crosscaps


disk
sphere w/ 1 hole
surfaces w/ boundary


cylinder
annulus
(sphere w/ 2 holes)


Sphere w/ 3 holes
(3 boundary components)
pair of pants





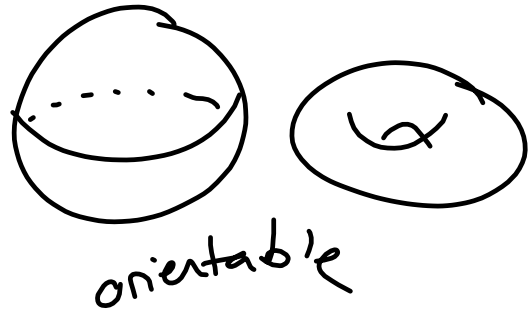
projective plane w/ hole (sphere w/ 1 crosscap + 1 hole) = Möbius band

The genus of a surface is the number of handles or crosscaps it has.

what if we have both?

In the presence of even one crosscap, every handle is equivalent to a crosscap.

Orientability



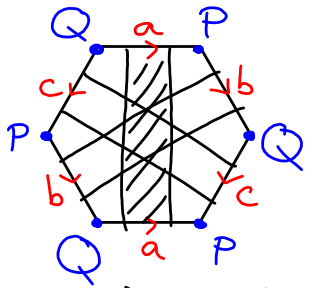
orientable

non-orientable

a twist gives us this

$abca^{-1}b^{-1}c^{-1}$

inverse letters have opposite orientation



no twists ⇒ orientable

1 face (1 word)
3 edges (a,b,c)
2 vertices (P,Q)

Euler characteristic

$\chi(S) = V - e + f$
$= 2 - 3 + 1 = 0$

$\chi(S) = 2 - 2g - b$
$\chi(S)$ = Euler char. = 0
g = genus
b = # of boundary components = 0

$0 = 2 - 2g - 0$
$2g = 2$
$g = 1$

orientable surface of genus 1 w/ no boundary components
sphere w/ 1 handle
torus

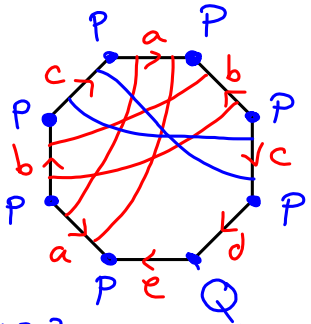
$a b^{-1} c d e \bar{a}^{-1} b c$

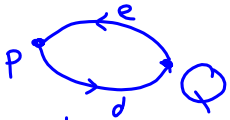
$f=1, e=5, v=2$

$\chi(S) = 2-5+1 = -2$

non-orientable

$\chi(S) = 2-g-b$





1 circle
$\Rightarrow$ 1 hole
$\Rightarrow b=1$

one boundary component

genus 3 nonorientable surface w/ 1 boundary component

$-2 = 2-g-1$

$g=3$

sphere w/ 3 crosscaps $\rightarrow$ 1 hole