Statement: If A, then B.

Contrapositive: If not B, then not A.

Statement: $x \in A \cup B \Rightarrow x \in A$ or $x \in B$
(If) (then)

Contrapositive:

If not ($x \in A$ or $x \in B$), then not ($x \in A \cup B$)

$x \notin A$ and $x \notin B \Rightarrow x \notin A \cup B$

10.

(a) $\{(x,y) \mid x \text{ is an integer}\} = \mathbb{Z} \times \mathbb{R}$
($\mathbb{Z}$: integers; $\mathbb{R}$: real #'s)

$\{(x,y) \mid x$ is a non-terminating, non-repeating decimal and $y \leq 3\}$

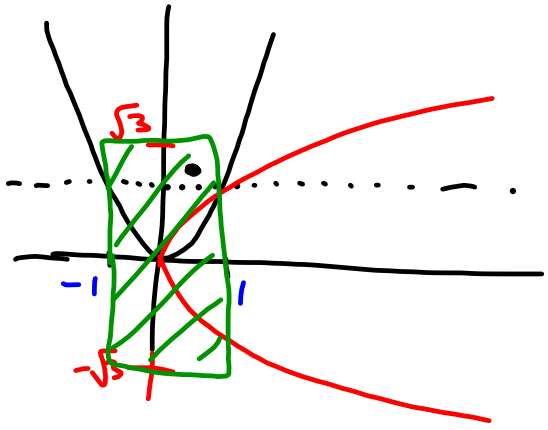
$(\mathbb{R} - \mathbb{Q}) \times (-\infty, 3]$
($\mathbb{R}$: reals; $\mathbb{Q}$: rationals; $\mathbb{R} - \mathbb{Q}$ = irrationals)

$$\{(x,y) \mid x^2 < 1 \text{ and } y^2 \leq 3\}$$

$(-1,1)$ $\quad [-\sqrt{3}, \sqrt{3}]$

$$\stackrel{?}{=} (-1,1) \times [-\sqrt{3}, \sqrt{3}]$$

true



$$A = \{(x,y) \mid x > y+3 \text{ and } y \text{ is an integer}\}$$

$\square \times \mathbb{Z}$

$(5,1) \in A$

$(7,3) \in A$

$(5,3) \notin A$ ← counter-example (not a cross product)

$x \in \mathbb{R} - \mathbb{Z}$
(x is not an integer)

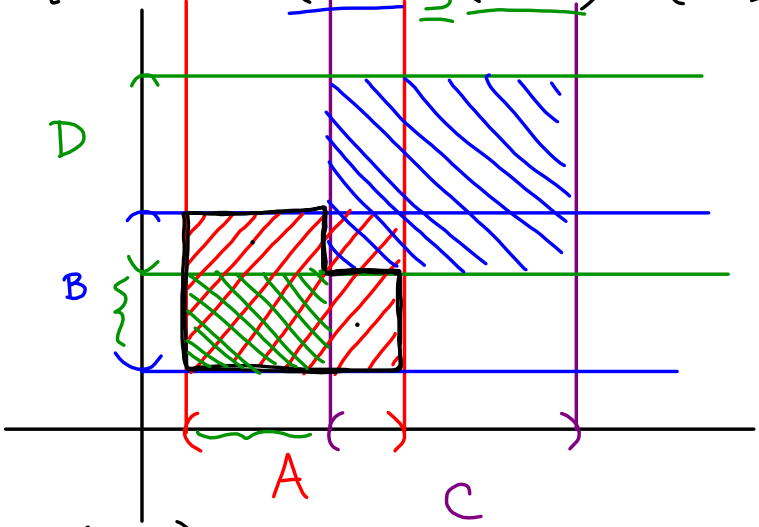
2q. $(A \times B) - (C \times D) = (A - C) \times (B - D)$

$\subseteq$: Let $(x,y) \in (A \times B) - (C \times D)$
$\Rightarrow (x,y) \in A \times B$ and $(x,y) \notin C \times D$
$\Rightarrow x \in A$ and $y \in B$ and $x \notin C$ or $x \notin D$

$\not\Rightarrow (x,y) \in (A-C) \times (B-C)$

$\supseteq$: Let $(x,y) \in (A-C) \times (B-D)$
$\Rightarrow x \in A-C$ and $y \in B-D$
. . .

2q. $(A \times B) - (C \times D) = (A - C) \times (B - D)$



$A = (1, 9)$, $C = (5, 20)$
$B = (1, 9)$, $D = (5, 20)$

$(7, 2) \in (A \times B) - (C \times D)$
but $(7,2) \notin (A-C) \times (B-D)$

$P(\{a\}) = \{\emptyset, \{a\}\}$

$P(\{a,b\}) = \{\emptyset, \{a,b\}, \{a\}, \{b\}\}$

$P(\{a,b,c\})$ . . .

$P(\emptyset) = \{\emptyset\}$

2e. $A-(A-B)=B$

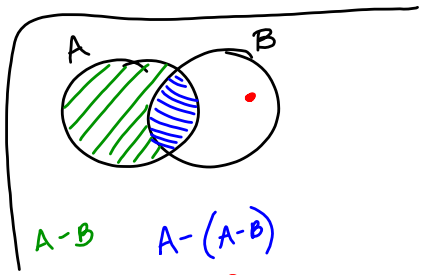
$\subseteq$: Let $x \in A-(A-B)$

$\Rightarrow x \in A$ and $x \notin (A-B)$

$\Rightarrow x \notin A$ or $x \in B$

Case 1:
$x \in A$ and $x \notin A$
$\Rightarrow x \in \emptyset \subseteq B$

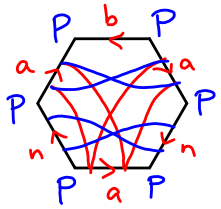
Case 2:
$x \in A$ and $x \in B$
$\Rightarrow x \in A \cap B \subseteq B$



Let $A=\{1,2\}$
$B=\{2,3\}$
$3 \in B$, but $3 \notin$

$A-(A-B) = \{1,2\} - (\{1,2\}-\{2,3\})$
$= \{1,2\} - \{1\}$
$= \{2\}$

$b^{-1}anā^{-1}na$ V: 1 (P)



E: 3 (b, a, n)

F: 1

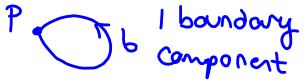
non-orientable

$\chi(S) = v - e + f$

$= 1 - 3 + 1 = -1$

$\chi(S) = 2 - g - b$

$-1 = 2 - g - 1$



$g = 2$

non-orientable surface of genus 2 w/ 1 boundary component

sphere w/ 2 crosscaps & 1 hole

$\cong$ Klein bottle w/ a hole