Given:

Collections of sets	$\mathcal{A},\ \mathcal{B},\ \mathcal{C}$
Sets	$A \in \mathcal{A},\ B \in \mathcal{B},\ C \in \mathcal{C}$
Subsets	$A_0 \subseteq A,\ B_0 \subseteq B,\ C_0 \subseteq C$
Elements	$a \in A_0,\ b \in B_0,\ c \in C_0$
Functions	$f: A \to B$ and $g: B \to C$

1. Which of the following is (not) necessarily a subset of set A?

   $\emptyset$ $A \cap B$ ($A \cup B$) $A - B$

2. Which of the following is set A (not) necessarily a subset of?

   $\emptyset$ $A \cap B$ ($A \cup B$) $A - B$

3. Given that $x \in A \cup B$, which of the following can we (not) assume?

   ($x \in A$ or $x \in B$); $x \in A \cap B$; $x \in C$

4. Given that $x \in A \cap B$, which of the following can we (not) assume?

   $x \in B$; $x \in A \cup B$; $x \in A \cup C$; ($x \in A \cap C$)

5. Given that $x \in A - B$, which of the following can we (not) assume?

   $x \in A$ (yes); $x \in B$ (not); $x \in A - (A \cap B)$ (yes); $x \in A \cup B$ (yes)

6. Given that $y \in f(A_0)$, which of the following can we (not) assume?

   $f^{-1}(y) \in A_0$ (yes); $y \in B$ (yes); $g(y) \in C$ (yes); $y \in B_0$ (no)

7. Given that $x \in f^{-1}(B_0)$, which of the following can we (not) assume?

   $f(x) \in B_0$

   $(g \circ f)(x) \in g(B_0)$ — yes

   $x \in f^{-1}(g(B_0))$ — no nonsense

   $g(x) \in g(B_0)$ — no nonsense

A function $f: A \to B$ is considered to be surjective (onto) if for every element $b \in B$, there exists an element $a \in A$ such that $f(a) = b$.

A function $f: A \to B$ is considered to be injective (one-to-one) if $f(x_1) = f(x_2)$ implies that $x_1 = x_2$ for all elements $x_1, x_2 \in A$.

. If a function $f: A \to B$ possesses both properties in the previous two questions, it is considered to be a bijection (one-to-one correspondence).

. Is the statement true

a. Always
b. Only if f is injective
c. Only if f is surjective
d. Never

$f(A_0 \cup A_1) \subseteq f(A_0) \cup f(A_1)$

$f(A \cap B) \subseteq f(A) \cap f(B)$

$f(A - B) \subseteq f(A) - f(B)$

f^{-1}

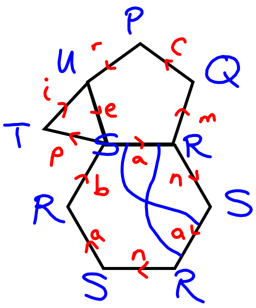
. Is the statement true

e. Always
f. Only if both f and g are injective
g. Only if both f and g are surjective
h. Never

$(g \circ f)(A \cup B$

(statements from 1.2 #1, 2, & 4)

b a n a n a c r e a m p i e



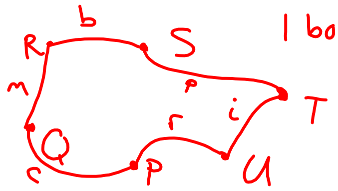
Faces: 3

Edges: b, a, n, c, r, e, m, p, i
9

Vertices: P Q R S T U
6

$\chi(S) = 6 - 9 + 3 = 0$

non-orientable



1 boundary component

$\chi(S) = 2 - g - b$

$0 = 2 - g - 1$

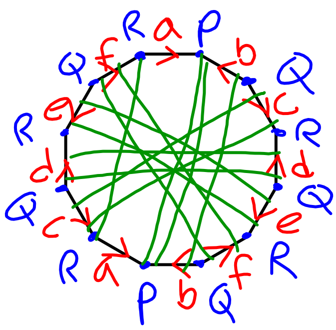
$g = 1$

non-orientable surface of genus 1 w/ 1 boundary comp.

⇒ a sphere w/ 1 crosscap & 1 hole

⇒ Möbius band!

$a b^{-1} c d^{-1} e f^{-1} b a^{-1} c^{-1} d e^{-1} f$



$F = 1$

$E = 6$

$V = 3$

$\chi(S) = 3 - 6 + 1$

$\chi(S) = -2$

$\chi(S) = 2 - 2g - b$

$-2 = 2 - 2g - 0$

$g = 2$

Orientable with genus 2 and no boundary components

Sphere with 2 handles and no holes

2-torus