Re: Final Presenations

- Double-check to make sure you did not schedule your presentation for a day that you will be taking an **AP exam**!
- You should be ready to present the day BEFORE your scheduled date just in case an emergency comes up and someone is unable to present; we can't afford to let any days go to waste.
- Don't forget that I need to see a draft of your paper/ presentation at least 3 school days prior to your presentation; this means I need to meet with Maggie no later than Tuesday, 4/29 and Emily no later than Wednesday, 4/30

Fri.	2 May	**The Product Topology** (Munkres 2.15)	Maggie Craft
Mon.	5 May	**The Subspace Topology** (Munkres 2.16)	Emily Cox
Tues.	6 May	**Connected Spaces** (Munkres 3.23-24)	Aaron Mullins
Wed.	7 May	**Compact Spaces** (Munkres 3.26)	Andrea Sibley
Fri.	9 May	**Space-Filling Curves** (Munkres 7.44, Armstrong 2.3)	Blake Sullivan
Mon.	12 May	**Homotopy of Paths** (Munkres 9.51, Armstrong 5.1, Hatcher Ch.0, 1.1)	Mellie Lammon
Tues.	13 May	**The Fundamental Group** (Munkres 9.52, Armstrong 5.2, Hatcher Ch.1)	Joanie Haas
Wed.	14 May	**The Brouwer Fixed-Point Theorem** (Munkres 9.55, Armstrong 5.5)	Justin Wahlers
Wed.	21 May	**The Quotient Topology** (Munkres 2.22)	Sung-Hoon Park
		Knots & Seifert Surfaces (Armstrong 10.1,10.3)	Ben Andrews

Closed Sets, Limit Points, and Continuity

Def Let X be a topological space. $A \subset X$ is a **closed set** if $X - A$ ia open.

Thm 17.1 Let X be a topological space. The following hold:

1) $\emptyset$, X are closed
2) arbitrary intersections of closed sets are closed, i.e. if A_i are closed, $\bigcap_i A_i$ is closed.
3) finite unions of closed sets are closed

Def Let X be a topological space and let $A \subset X$. The **dosure** of A, denoted by $\bar{A}$, is the intersection of all closed sets containing A.

Thm 17.5 Let $A \subset X$and let $\mathfrak{B}$ be a basis for X. Then $x \in \bar{A}$ if and only if every open set U containing x intersects A and $x \in \bar{A}$ if and only if every basis element B containing x intersects A

Def U is a **neighborhood** of x if U is open and $x \in U$

Def Let X be a topological space and let $A \subset X$. $x \in X$ is said to be a **limit point** of A if every neighborhood of x intersects A in a point other than x.

Thm 17.6 Let $A \subset X$ and let A' be the set of all limit points of A. Then $\bar{A} = A \cup A'$.

Cor 17.7 A is closed if and only if A contains all its limit points.

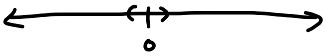
Def A topological space is **Hausdorff** if for each pair of distinct points x and y, there exist disjoint neighborhoods of x and y.

Thm 17.8 Every finite set in a Hausdorff space is closed.* finite union of single point sets

Thm 17.9 Let X be a Hausdorff space and let $A \subset X$. Then x is a limit point of A if and only if every neighborhood of x contains infinitely many points of A.

*finite sets being closed is weaker condition than Hausdorff, and is called the T_1-axiom

e.g. the set of positive integers with the co-finite topology satisfies the T_1-axiom but is not Hausdorff



X-Hausdorff space ; $A = \{a\}$, $A \subseteq X$

$X - A = X - \{a\}$ Let $b \in X$.

Since X is Hausdorff, $\exists$ open neighborhoods U & V in X s.t. $U \cap V = \emptyset$, $a \in U$, $b \in V$.

$V \subseteq X - \{a\} \Rightarrow b \in X - \{a\}$

open

Given $b \in X - \{a\}$ $\exists$ V open in X s.t. $b \in V$ and $V \subseteq X - \{a\}$.

$\Rightarrow X - \{a\}$ is open in X and hence $\{a\}$ is closed in X.

Def Let X and Y be topological spaces and $f: X \to Y$ be a function. f is said to be **continuous** if for every open set V in Y, $f^{-1}(V)$ is open in X.
$f^{-1}(V) = \{x \in X | f(x) \in V\}$

If $\mathcal{B}$ is a basis for Y, then f is continuous if $f^{-1}(B)$ is open for all $B \in \mathcal{B}$.

Examples:

1. $f: X \to Y$, $f(x) = y_0 \in Y$ for all $x \in X$
(constant function)

V - open set in Y

$$f^{-1}(V) = \begin{cases} X & , \text{if } y_0 \in V \\ \emptyset & , \text{if } y_0 \notin V \end{cases}$$

since X and $\emptyset$ are open in X, f is continuous.

2. $\pi_1 : X \times Y \longrightarrow X$

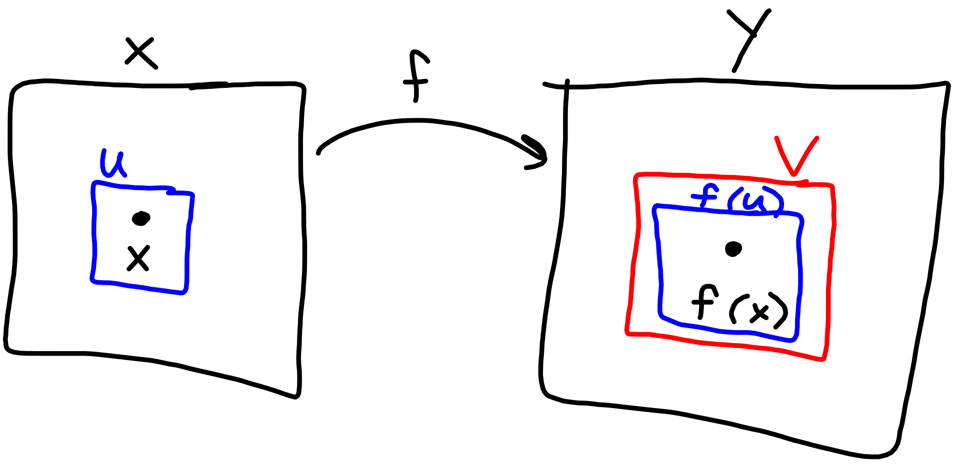
$\pi_1(x,y) = x$ projection onto first coordinate

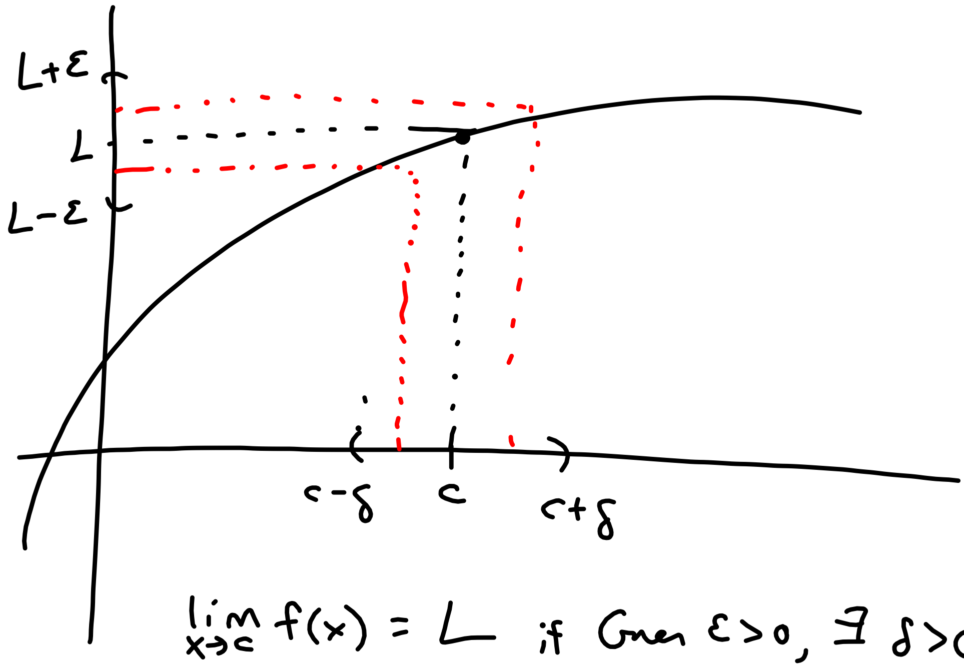
V - open set in X

$\pi_1^{-1}(V) = V \times Y$ cartesian product of 2 open sets is open

Thm 18.1 Let X and Y be topological spaces and $f: X \to Y$ be a function. The following are equivalent:

1) f is continuous
2) $f^{-1}(B)$ is closed for every closed set $B \subset Y$
3) for each $x \in X$ and each neighborhood V of $f(x)$, there exists a neighborhood U of x such that $f(U) \subset V$.





$\lim_{x\to c} f(x) = L$ if Given $\varepsilon>0$, $\exists\ \delta>0$ s.t. $|f(x)-L|<\varepsilon$ whenever $|x-c|<\delta$.

f is cts if $f(c)=L$.

Def Let $f: X \to Y$ be a bijection. If both f and f^{-1} are continuous, then f is called a **homeomorphism**.