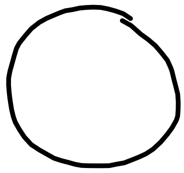
A knot is a simple closed curve in Euclidean 3-D space. Basically, they are different projections of a circle into three-dimensional space.

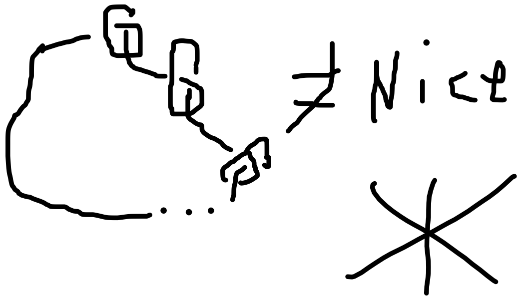
Ex: The Trivial Knot or Unknot



Def: Two knots k_1, and k_2 are equivilent if there exists a homeomorphism h *s.t.* $h(k_1)=k_2$

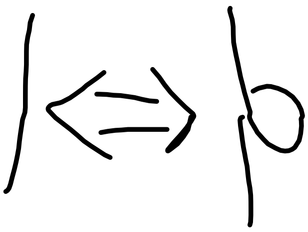
Given the knot, is there a homeomorphism that maps it to the unknot?

Nice Knots- knots that cross a finite number of times, with no crosses on boundaries or more than two strands crossing in one place.

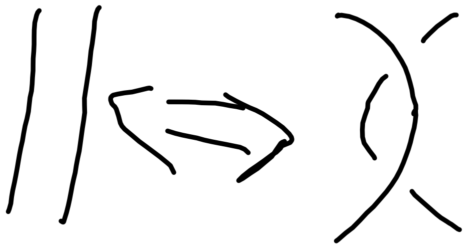


Reidemeister Moves- all moves one could use to translate one knot to another are just a combination of three simple ones- Proven by Kurt Reidemeister

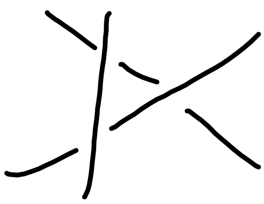
1.Twist or untwist a loop

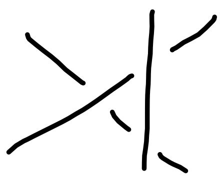


2. Move one strand over another



3. Move a stand above or beneath a crossing.

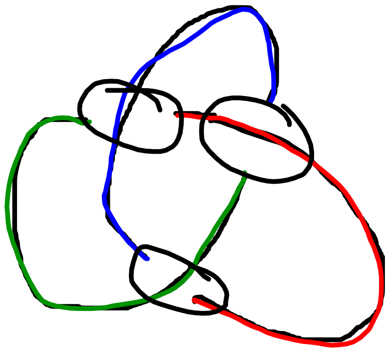




Certain features of knots do (k)not vary in different projections. These are called Knot Invariants

- Minimal Crossing Number- how many times a knot crosses itself in its simplest form
- Unknotting Number- the number of crosses that would have to be changed to create the unknot
- Tricolorability- If you designate colors to sections of a knot, using at least 2 colors, the crossing sections will be the intersection of strands of all the same color or strands of only different colors.

Let us attempt to find these invariants on the trefoil knot.

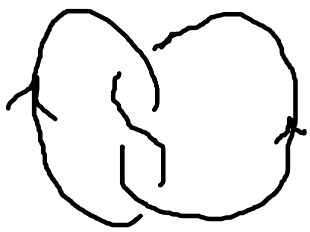


Min Cross # = 3
Unknot # = 1
Tricolor = Yes

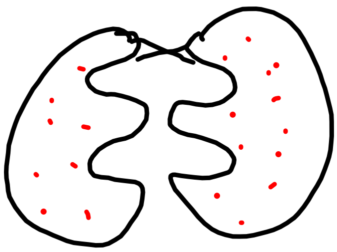
Def: A seifert surface is a disk with a tame knot as its boundary component.

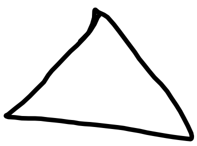
Tame Knots- these knots are knots that can be made up of a finite number of line segments.

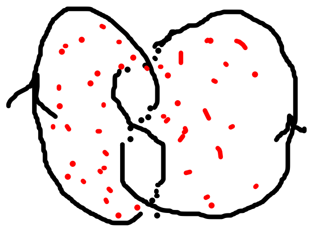
1



← Creating a Seifert Surface



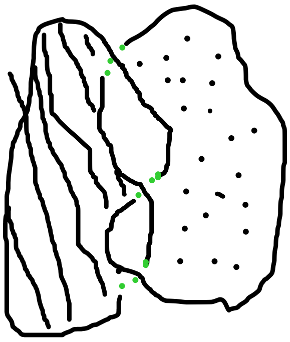




It is best to use a projection such as this of the trefoil as a seifert surface is most easily created by finding disjoint oriented circles in the knot, spanning them by a disk, annd connecting them with twists to represent the crossings and to orient them.

Genus of a Seifert Surface

The genus of a Seifert Surface can be found by $g=(c-s+1)/2$, where g is the genus, c is the number of crossings, and s is the number of seifert surfaces.



Note The genus of a seifert surface for a knot varies depending on the projection of the knot.

$c = 3$

$s = 2$

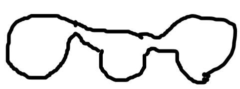
$g=(c-s+1)/2$

$g=(3-2+1)/2$

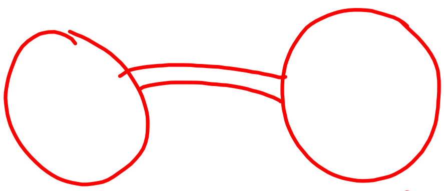
$g=1$

Surgery connects two Seifert Surfaces together by simply connecting them with a strip away from the knot

$g_1=1$ $g_2=0$ $g_1+g_2=1$



A tame ~~knot~~ surface/disc is equivalent to a normal unit disc or a unit disc with more discs equivalent to the unit disc connected by surgery.



$g=0$ $g=0$

Theorem: A knot is equivilent to the trivial knot if and only if it can be spanned by a tame disc.

Suppose K is equivilent to the unknot and let h be a homeomorphism which throws k onto the boundary of the unit disc D. $h^{-1}(D)$ is a tame disc spanning K.

If we a polygonal knot spanned by a unit disk embedded polygonally