1.1 - Set Theory

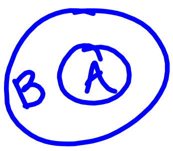
A - capital letters for sets

a - lower case letters for elements of sets

$a \in A$ "a is an element of the set A"

Subsets

or



Munkres

$A \subset B$
subset
(A lives inside B or might be equal to B)

$A \subsetneq B$
proper subset
(A lives inside B, but $A \neq B$)

$A \subseteq B$ $\qquad$ $A \subset B$

$A = B$ implies $A \subseteq B$ and $B \subseteq A$

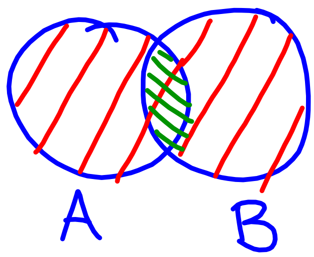
$A \subset B \Rightarrow B \supset A$

"A is a subset of B" "B contains A"

union / intersection

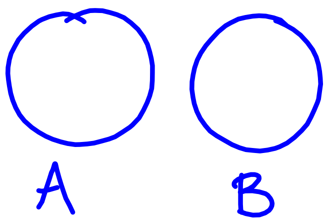
$\underline{A \cup B} = \{x \mid x \in A \text{ or } x \in B\}$

$\underline{A \cap B} = \{x \mid x \in A \text{ and } x \in B\}$



$A \subseteq A \cup B$

$A \cap B \subseteq A$



$A \cap B = \emptyset$ = the set containing no elements

$\emptyset \ ? \ A$

$A \subseteq B \implies$ If $x \in A$, then $x \in B$.

For $\emptyset \subset A$, we need:

If $x \in \emptyset$, then $x \in A$.

"vacuously true"

for any set A

Formal Logic "If... then..." statements.

Statement: If P, then Q. $P \Rightarrow Q$ "P implies Q"

"If it's raining, then the streets are wet."

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contrapositive: If (not Q), then (not P).

"If the streets are not wet, then it is not raining." $\sim Q \Rightarrow \sim P$

Logically equivalent to original statement

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converse: If Q, then P. $Q \Rightarrow P$

"If the streets are wet, then it is raining."

NOT logically equivalent to original statement

Statement:

For every $x \in A$, statement P holds.
If $x \in A$, then P holds.

$\forall x \in A$, P holds
"for all"

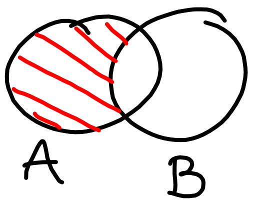
Negation of this statement?

There exists some (at least one) $x \in A$ for which P does not hold.

$\exists x \in A$ s.t. $\sim P$.

Difference of Sets:

$$A - B = \{x \mid x \in A \text{ but } x \notin B\}$$



Distributive Properties:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

De Morgan's Laws:

$$A - (B \cup C) = (A - B) \cap (A - C)$$
$$A - (B \cap C) = (A - B) \cup (A - C)$$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof:

$\subseteq$: Let $x \in A \cap (B \cup C)$.

To show that $x \in (A \cap B) \cup (A \cap C)$.

$x \in A \cap (B \cup C) \Rightarrow$

$x \in A$ and $x \in B \cup C$.

$x \in B \cup C \Rightarrow x \in B$ or $x \in C$.

case 1: $x \in A$ and $x \in B$

$\Rightarrow x \in A \cap B$

case 2: $x \in A$ and $x \in C$

$\Rightarrow x \in A \cap C$