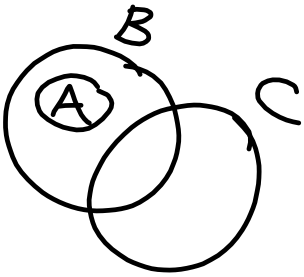
2a. $A \subset B$ and $A \subset C \iff A \subset (B \cup C)$ false

"if and only if"

$A \subset (B \cap C) \underset{\text{true}}{\Longrightarrow} A \subset B$ or $A \subset C$

Proof : $A \subset B$ and $A \subset C$

$B \subset (B \cup C) \Rightarrow A \subset (B \cup C)$



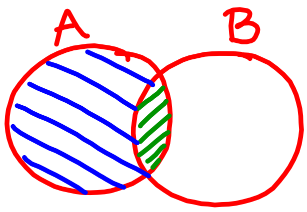
$\overset{?}{\Longleftarrow}$ false

e.g. $A \subset (B - C)$

(or $A \subset (C - B)$)

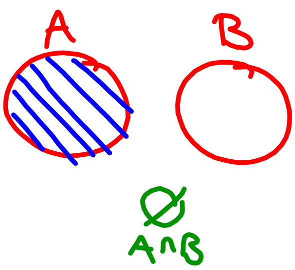
2i. $(A \cap B) \cup (A - B) = A$ True

$\overset{?}{\subseteq}$, $\overset{?}{\supseteq}$ true



If $A = \emptyset$?

If $B = \emptyset$?



Cross-Product of Sets A and B

$$A \times B = \{(a,b) \mid a \in A,\ b \in B\}$$

2m. $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$

(a,b) $\quad$ (c,d)

$\overset{?}{\subseteq}$ true

$\overset{?}{\supseteq}$ false

(a,b), (a,d), (c,b), (c,d)

$\{(x,y) \mid x \in A$ or $x \in C$ and $y \in B$ or $y \in D\}$

2m. $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$

$(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$

<u>Proof</u>: Let $(x,y) \in (A \times B) \cup (C \times D)$.

$\Rightarrow (x,y) \in A \times B$ or $(x,y) \in (C \times D)$.

Case 1: $(x,y) \in A \times B \Rightarrow x \in A$ and $y \in B$

$x \in A$ and $A \subseteq A \cup C \Rightarrow x \in A \cup C$

$y \in B$ and $B \subseteq B \cup D \Rightarrow y \in B \cup D$

$x \in A \cup C$ and $y \in B \cup D \Rightarrow (x,y) \in (A \cup C) \times (B \cup D)$.

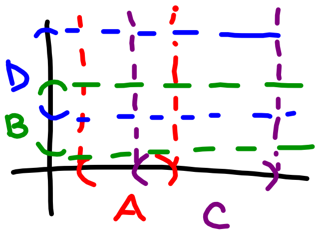
Case 2: $(x,y) \in C \times D$ (proof similar to case 1) $\square$

2m. $(A\times B)\cup(C\times D)=(A\cup C)\times(B\cup D)$

$$\underset{(a,b)}{(A\times B)}\cup\underset{(c,d)}{(C\times D)} \not\supseteq (A\cup C)\times(B\cup D)$$

(a,b) (c,b)
(a,d) (c,d)

Proof: Note: to show that a statement is false, we need only to produce a single counter-example.



In this case, some ordered pair (x,y) that lies in $(A\cup C)\times(B\cup D)$ but not in $(A\times B)\cup(C\times D)$.

$A=\emptyset$
$B=\{1\}$
$C=\{2\}$
$D=\{3\}$

$(2,1)\in C\times B\subset(A\cup C)\times(B\cup D)$
but $(2,1)\notin A\times B$
$\notin C\times D$