$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$\subseteq$: Let $x \in A \cup (B \cap C)$.

Case 1 : $x \in A$

$x \in A$ and $A \subseteq A \cup B$ and $A \subseteq A \cup C$

$\Rightarrow x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

Case 2: $x \in (B \cap C)$

$\Rightarrow x \in B$ and $x \in C$

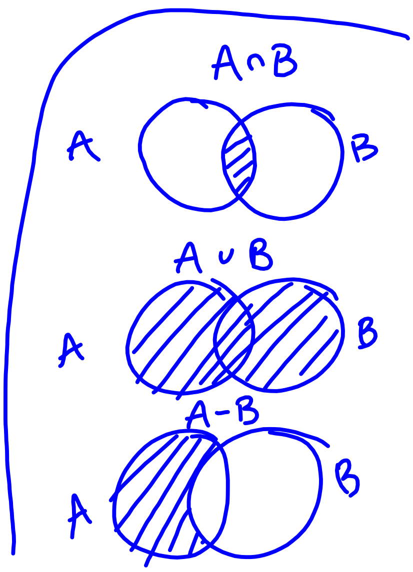
$B \subseteq A \cup B$ and $x \in B \Rightarrow x \in A \cup B$

$C \subseteq A \cup C$ and $x \in C \Rightarrow x \in A \cup C$

$x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$.

Hence $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.



$x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$

$x \in A \cap B \Rightarrow x \in A$ and $x \in B$

$x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$

$x \in A - B \Rightarrow x \in A$ and $x \notin B$

$x \notin A - B \Rightarrow x \notin A$ or $x \in B$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$\supseteq$: Let $y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B)$ and $y \in (A \cup C)$

$\Rightarrow (y \in A$ or $y \in B)$ and $(y \in A$ or $y \in C)$

4 cases: $y \in A$ and $y \in A$

$y \in A$ and $y \in C$

$y \in B$ and $y \in A$

$\Rightarrow y \in A$ and $A \subseteq A \cup (B \cap C)$

$\Rightarrow y \in A \cup (B \cap C)$

$y \in B$ and $y \in C \Rightarrow y \in B \cap C$ and $(B \cap C) \subseteq A \cup (B \cap C)$

$\Rightarrow y \in A \cup (B \cap C)$

Hence $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

and so $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. $\square$

$A - (B \cup C) = (A - B) \cap (A - C)$

$\subseteq$: Let $x \in A - (B \cup C)$

$\Rightarrow x \in A$ and $x \notin B \cup C$

$\Rightarrow x \in A$ and $x \notin B$ $\boxed{\text{and}}$* $x \notin C$

$x \in A$ and $x \notin B \Rightarrow x \in A - B$

$x \in A$ and $x \notin C \Rightarrow x \in A - C$

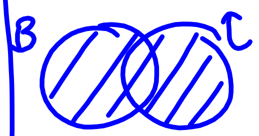
$x \in A - B$ and $x \in A - C \Rightarrow$

$x \in (A - B) \cap (A - C) \Rightarrow$

$A - (B \cup C) \subseteq (A - B) \cap (A - C)$.

$x \in B \cup C$

$\Rightarrow x \in B$ or $x \in C$



$x \notin B \cup C$

$\Rightarrow$

$x \notin B$ and $x \notin C$

$$A-(B\cup C)=(A-B)\cap(A-C)$$

$\supseteq$: Let $y\in(A-B)\cap(A-C)$

$\Rightarrow y\in A-B$ and $y\in A-C$

$\Rightarrow y\in A$ and $y\notin B$ and $y\in A$ and $y\notin C$

$y\notin B$ and $y\notin C \Rightarrow y\notin B\cup C$

$y\in A$ and $y\notin B\cup C \Rightarrow y\in A-(B\cup C)$.

$\Rightarrow (A-B)\cap(A-C)\subseteq A-(B\cup C)$

Hence $A-(B\cup C)=(A-B)\cap(A-C)$. $\square$

$$A-(B\cap C)=(A-B)\cup(A-C)$$

$\subseteq$: Let $x\in A-(B\cap C)$

$\Rightarrow x\in A$ and $x\notin B\cap C$

$\Rightarrow x\in A$ and $(x\notin B$ or $x\notin C)$

Case 1: $x\in A$ and $x\notin B \Rightarrow x\in A-B$

$A-B\subseteq(A-B)\cup(A-C) \Rightarrow x\in(A-B)\cup(A-C)$.

Case 2: $x\in A$ and $x\notin C \Rightarrow x\in A-C$

$A-C\subseteq(A-B)\cup(A-C)\Rightarrow x\in(A-B)\cup(A-C)$.

Hence $A-(B\cap C)\subseteq(A-B)\cup(A-C)$.

$$A-(B\cap C)=(A-B)\cup(A-C)$$

$\supseteq$: Let $y\in(A-B)\cup(A-C)$.

$\Rightarrow y\in A-B$ or $y\in A-C$

Case 1: $y\in A-B \Rightarrow y\in A$ and $y\notin B$

$y\notin B$ and $B\cap C\subseteq B \Rightarrow y\notin B\cap C$

$y\in A$ and $y\notin B\cap C \Rightarrow y\in A-(B\cap C)$.

Case 2: $y\in A-C$

similar