Statement: $A \Rightarrow B$
"A implies B"
If A, then B

Contrapositive: If not B, then not A
$\sim B \Rightarrow \sim A$
* logically equivalent to original statement

Converse: If B, then A
$B \Rightarrow A$

$\Longleftrightarrow$ If and only if

$A \Leftrightarrow B$ $\quad A \Rightarrow B$ and $B \Rightarrow A$

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statement:
If $x^2 > 0$, then $x > 0$.
false: $(-2)^2 = 4 > 0$, but $-2 < 0$.

contrapositive:
If $x \leq 0$, then $x^2 \leq 0$.
false (same counter-example works)

Converse:
If $x > 0$, then $x^2 > 0$.
true

Statement :

For all $x \in A$, $x \in A \cap B$.

Negation:

There exists at least one $x \in A$

such that $x \notin A \cap B$.

$$\boxed{\exists\, x \in A \text{ such that } x \notin A \text{ or } x \notin B.}$$

Statement :

There exists $x \in \mathbb{R}$ such that

$\sqrt{x} = \pi$.

Negation:

For all $x \in \mathbb{R}$, $\sqrt{x} \neq \pi$.

The power set of a set X is the set of all subsets of X.

$X = \{1,2,3,4,5\}$

$\mathcal{P}(X) = \{ \emptyset, X, \{1\}, \{2\}, \{3\}, \{4\} \{5\},$
$\{1,2\}, \{1,3\}, \ldots, \{2,3\}, \ldots \{3,4\}. \{4,5\}$
$\{1,2,3\} \ldots$
$\{1,2,3,4\} \ldots \}$

this power set contains 2^5 sets as elements

$A_0 = \emptyset$
$\mathcal{P}(A_0) = \{\emptyset\}$

$A_1 = \{a\}$
$\mathcal{P}(A_1) = \{\emptyset, \{a\}\}$

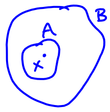
$A_2 = \{a, b\}$
$\mathcal{P}(A_2) =$

$A_3 = \{a, b, c\}$
$\mathcal{P}(A_3) =$

Cartesian product

$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$

$A \subseteq B$ if $x \in A$ implies $x \in B$.



1.1

2.m.

$(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$

$\subseteq$: Let $(x,y) \in (A \times B) \cup (C \times D)$.

$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in (C \times D)$.

Case 1: $(x,y) \in (A \times B) \Rightarrow x \in A$ and $y \in B$.

$$\left.\begin{array}{l} A \subseteq A \cup C \Rightarrow x \in A \cup C \\ B \subseteq B \cup D \Rightarrow y \in B \cup D \end{array}\right\} \Rightarrow (x,y) \in (A \cup C) \times (B \cup D).$$

Case 2 similar.

$\supseteq$: Let $(x,y) \in (A \cup C) \times (B \cup D)$.

$\Rightarrow x \in A \cup C$ and $y \in B \cup D$.

$\Rightarrow (x \in A \text{ or } x \in C)$ and $(y \in B \text{ or } y \in D)$

Case 1: $x \in A$ and $y \in B \Rightarrow (x,y) \in (A \times B) \subseteq (A \times B) \cup (C \times D)$

$$\left.\begin{array}{l} \text{Case 2: } x \in A \text{ and } y \in D \\ \text{Case 3: } x \in C \text{ and } y \in B \end{array}\right\} \quad \supseteq \text{ is false}$$

Case 4: $x \in C$ and $y \in D \Rightarrow (x,y) \in (C \times D) \subseteq (A \times B) \cup (C \times D)$

$(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$

counter-example for $\supseteq$:

$A = \{1\}$ $\quad A \times B = \{(1,2)\}$ $\quad A \cup C = \{1,3\}$

$B = \{2\}$ $\quad C \times D = \{(3,4)\}$ $\quad B \cup D = \{2,4\}$

$C = \{3\}$ $\quad (A \cup C) \times (B \cup D) =$

$D = \{4\}$ $\quad (A \times B) \cup (C \times D) = \{(1,2),(3,4)\}$ $\quad \{(1,2),(3,4),(1,4),(3,2)\}$

$(1,4) \in (A \cup C) \times (B \cup D)$, but $(1,4) \notin (A \times B) \cup (C \times D)$. $\square$

$x \in A - B \Rightarrow x \in A$ and $x \notin B$

$x \notin A - B \Rightarrow x \notin A$ or $x \in B$

$(x,y) \in A \times B \Rightarrow x \in A$ and $x \in B$

$(x,y) \notin A \times B \Rightarrow x \notin A$ or $x \notin B$