# Munkres 1.1

## Set Theory

$A$ set

$a$ element of a set

$a \in A$

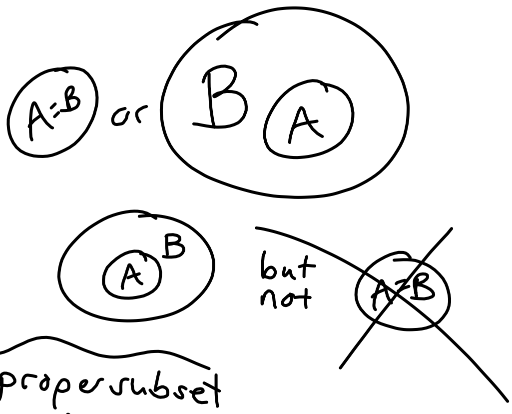
$\in$ means "is an element of"

subsets

$A \subset B$

$A \subsetneq B$

proper subset



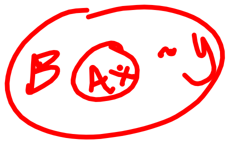

$\subseteq$ subset

$\subset$ proper sub

subset $A \subseteq B$

proper subset $A \subset B$

If $A \subseteq B$, then

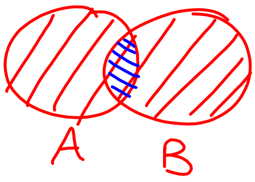



if $x \in A$, then $x \in B$.

but if $y \in B$, then $y$ is not necessarily in $A$.

union & intersection

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$




union

$A \subseteq A \cup B$

$B \subseteq A \cup B$

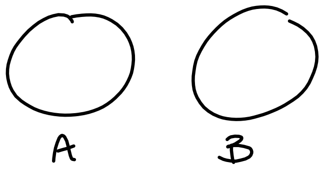
$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

intersection

$A \cap B \subseteq A$

$A \cap B \subseteq B$

$A \cap B \subseteq A \cup B$




$\{\emptyset\} \neq \emptyset$

$A \cap B = ? = \emptyset$

$\emptyset$ = "empty set" =

= set containing no elements

$A \subset B$

If $x \in A$, then $x \in B$.

$\emptyset \subset A$ "vacuously true" for any set $A$

$A \cap \emptyset = \emptyset$

$A \cup \emptyset = A$

$A = B$

$\Rightarrow A \subseteq B$ and $B \subseteq A$

Case 1 $A \subseteq B$

Assume $x \in A$. To show $x \in B$

$\vdots$

Case 2. $B \subseteq A$

Assume $y \in B$. To show $y \in A$.

# Formal Logic

If A, then B. $\qquad A \rightarrow B$

(If A is true, then B is true.) If it is raining, then the streets are wet.

$A \Rightarrow B$ "A implies B"

contrapositive: If (not B) then (not A). $\qquad \sim B \rightarrow \sim A$

If the streets are not wet, then it is not raining. $\sim B \Rightarrow \sim A$

Logically equivalent to original statement

converse: If B, then A. $B \Rightarrow A$ $\qquad B \rightarrow A$

If the streets are wet, then it is raining.

NOT logically equivalent to original statement

$A \Leftrightarrow B$

$A \Rightarrow B$ and $B \Rightarrow A$

$A$ if and only if $B$

Thm: For every $\underbrace{x \in A}_{\text{If } A}$, $\underbrace{\text{statement } P \text{ holds}}_{\text{then } B}$.

$\forall$ $x \in A$, statement $P$ holds.
"for all"

Contrapositive: if $\sim B$ then $\sim A$

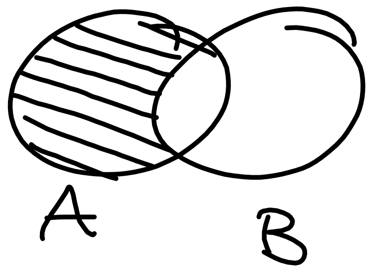
If statement $P$ does not hold, then [not every $x$ is an element of $A$] — there exists

$\exists$ $x \in A$ such that statement $P$ does not hold.
"there exists"

## Difference of Sets

(Relative complement)

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$




$x \notin A - B$ if $x \notin A$ or $x \in B$

Set theory rules

"Distributive Properties"

* $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De Morgan's Laws:

$A - (B \cup C) = (A - B) \cap (A - C)$

$A - (B \cap C) = (A - B) \cup (A - C)$