If $x^2 < 0$, then $x = 23$.

If $x \neq 23$, then $x^2 \geq 0$.

Set theory rules

"Distributive Properties"

* $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

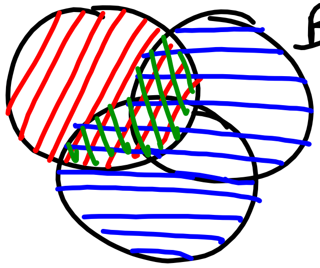
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

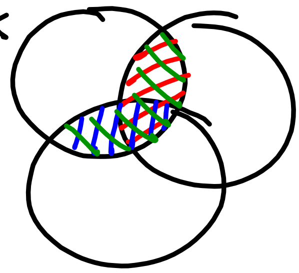
De Morgan's Laws:

$A - (B \cup C) = (A - B) \cap (A - C)$

$A - (B \cap C) = (A - B) \cup (A - C)$

Theorem: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$





Theorem: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof $(\subseteq)$

To show that $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C)$.

Then $x \in A$ and $x \in B \cup C$.

Then $x \in A$ and either $x \in B$ or $x \in C$.

Case 1: $x \in A$ and $x \in B$

$\Rightarrow x \in A \cap B$. Since $A \cap B \subseteq (A \cap B) \cup (A \cap C)$ we have that $x \in (A \cap B) \cup (A \cap C)$.

Case 2: $x \in A$ and $x \in C$.

$\Rightarrow x \in A \cap C$. Since $A \cap C \subseteq (A \cap B) \cup (A \cap C)$ we have $x \in (A \cap B) \cup (A \cap C)$.

Theorem: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

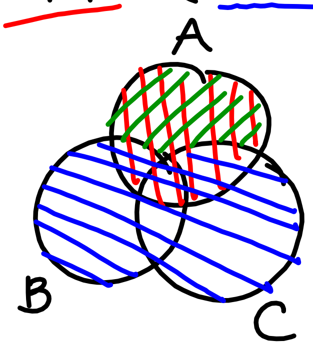
$\supseteq$: Let $y \in (A \cap B) \cup (A \cap C)$.

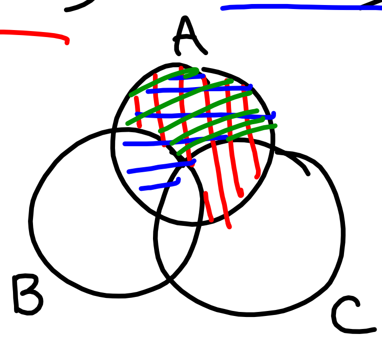
$\Rightarrow$ either $y \in A \cap B$ or $y \in A \cap C$.

Case 1: $y \in A \cap B \Rightarrow y \in A$ and $y \in B$.
Since $B \subseteq B \cup C$, $y \in B \cup C$.
Since $y \in A$ and $y \in B \cup C$,
we have $y \in A \cap (B \cup C)$.

Case 2: $y \in A \cap C \Rightarrow y \in A$ and $y \in C$.
Since $C \subseteq B \cup C$, $y \in B \cup C$.
Since $y \in A$ and $y \in B \cup C$,
we have $y \in A \cap (B \cup C)$. $\square$

$A - (B \cup C) = (A - B) \cap (A - C)$





$$A-(B\cup C)=(A-B)\cap(A-C)$$

Proof: $\subseteq$

Let $x\in A-(B\cup C)$.

$\Rightarrow x\in A$ and $x\notin B\cup C$.

$\Rightarrow x\in A$ and $x\notin B$ and $x\notin C$.

$x\in A$ and $x\notin B \Rightarrow x\in A-B$.

$x\in A$ and $x\notin C \Rightarrow x\in A-C$.

$\Rightarrow x\in (A-B)\cap(A-C)$.

$$A-(B\cup C)=(A-B)\cap(A-C)$$

$\supseteq$: Let $y\in(A-B)\cap(A-C)$.

$\Rightarrow y\in A-B$ and $y\in A-C$.

$y\in A-B \Rightarrow y\in A$ and $y\notin B$.

$y\in A-C \Rightarrow y\in A$ and $y\notin C$.

$y\notin B$ and $y\notin C \Rightarrow y\notin B\cup C$.

$y\in A$ and $y\notin B\cup C \Rightarrow y\in A-(B\cup C)$. $\square$

A - capital letters for sets

a - lowercase letters for elements of set

$\mathcal{A}$ - script capitals for collections of sets

"Power Set" of set A : $\mathcal{P}(A)$

is the set of all of the subsets of A.

$A = \{a, b, c, d\}$

$\mathcal{P}(A) = \{ \emptyset, A, \{a,b\}, \{a,b,c\}, \{b\}, \{a\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\} \}$

$(\{a,b,c,d\})$

If A contains n elements,
$\mathcal{P}(A)$ will contain 2^n elements.

$$\bigcup_{A \in \mathcal{A}} A = \{x \mid x \in A \text{ for some } A \in \mathcal{A}\}$$
(at least one)

$$\bigcap_{A \in \mathcal{A}} A = \{x \mid x \in A \text{ for all } A \in \mathcal{A}\}$$
$x \in A \; \forall \; A \in \mathcal{A}$

$\bigcap_{A \in \mathcal{A}} A \subseteq A$ for all A

$A \subseteq \bigcup_{A \in \mathcal{A}} A$ for all A

Regular De Morgan

$$A-(B\cap C) = (A-B)\cup(A-C)$$

De Morgan's Law for arbitrary intersections

$$A - \bigcap_{B\in\mathcal{B}} B = \bigcup_{B\in\mathcal{B}} (A-B)$$

$x \in A - \bigcap_{B\in\mathcal{B}} B \Rightarrow x\in A$ and $x \notin \bigcap_{B\in\mathcal{B}} B$

$x \notin \bigcap_{B\in\mathcal{B}} B \Rightarrow x \notin B$ for at least one $B \in \mathcal{B}$.

§1.1

2a. $A\subset B$ and $A \subset C \Leftrightarrow A \subset (B\cup C)$

$\Rightarrow A \subset B\cap C \subset B\cup C \Rightarrow A \subset (B\cup C)$ True

$\Leftarrow A\subset (B\cup C) \Rightarrow A\subset B$ or $A\subset C$

False: $A \subset B$ and $A \not\subset C$.

$\Leftrightarrow$ False.

2i. $(A \cap B) \cup (A - B) = A$