Cartesian Product

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

$\mathbb{R} \times \mathbb{R}$ = 2-dim'l cartesian plane of (x,y) coordinates

2m. $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$

$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$

$C \times D = \{(c,d) \mid c \in C \text{ and } d \in D\}$

$A \cup C = \{x \mid x \in A \text{ or } x \in C\}$

$B \cup D = \{y \mid y \in B \text{ or } y \in D\}$

$\supseteq$: (a,d), $a \in A$, $d \in D$

false $\quad (a,d) \in (A \cup C) \times (B \cup D)$

$(a,d) \notin (A \times B) \cup (C \times D)$

$\subseteq$: Let $(x,y) \in (A \times B) \cup (C \times D)$

$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in (C \times D)$

$\Rightarrow x \in A$ and $y \in B$ or $x \in C$ and $y \in D$

case 1: if $x \in A$ and $y \in B$

since $A \subseteq A \cup C$, $x \in A \cup C$

$B \subseteq B \cup D$, $y \in B \cup D$

$\Rightarrow (x,y) \in (A \cup C) \times (B \cup D)$.

case 2 is similar.

conditional statement

If $x>0$, then $x^3 \neq 0$. $A \to B$

true

converse:

If $x^3 \neq 0$, then $x>0$. $B \to A$

false: $x^3=-1$ but $x<0$

contrapositive: $\sim B \to \sim A$

If $x^3=0$, then $x \leq 0$ true

$\lim_{x \to a} f(x)=L$ if for every $\varepsilon>0$,

there exists $\delta>0$ such that

$|x-a|<\delta$ implies $|f(x)-L|<\varepsilon$. $A \to B$

contrapositive $\sim B \to \sim A$

If $\lim_{x \to a} f(x) \neq L$, then there exists

$\varepsilon>0$ such that for any $\delta>0$

such that $|x-a|<\delta$ but $|f(x)-L| \geq \varepsilon$

2b. $A \subset B$ or $A \subset C \Leftrightarrow A \subset (B \cup C)$

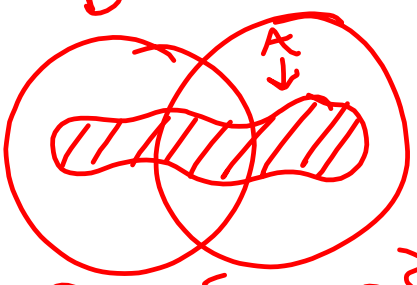
false

$\Rightarrow$ definition true

$\Leftarrow$ ~~definition~~ false

$A = \emptyset$ ✓

$A \neq \emptyset$



$B = \{1, 2\}$ $C = \{2, 3\}$

$A = \{1, 3\}$

$B \cup C = \{1, 2, 3\}$

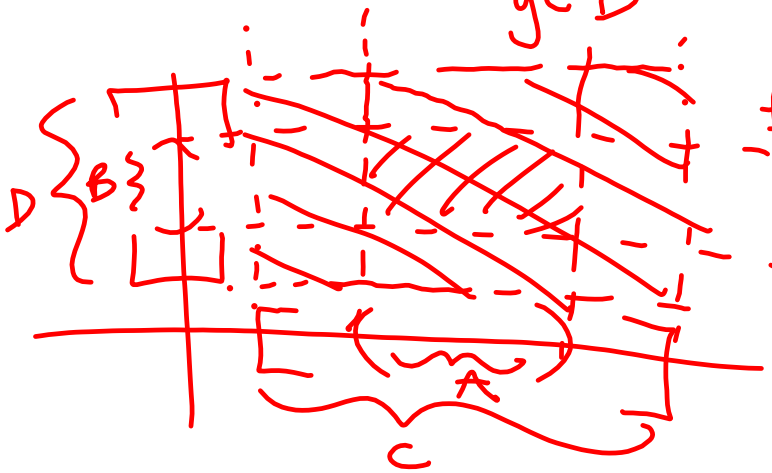
j. $A \subset C$ and $B \subset D \Rightarrow (A \times B) \subset (C \times D)$

If $x \in A$, then $x \in C$

If $y \in B$, then $y \in D$

If $(x, y) \in A \times B$, then $(x, y) \in C \times D$

If $x \in A$ and $y \in B$, then $x \in C$ and $y \in D$.



K. $A \subset C$ and $B \subset D \Leftarrow (A \times B) \subset (C \times D)$

false

$A = \{1, 2, 3\}$

$B = \emptyset$

$C = \{4, 5, 6\}$

$D = \{7, 8, 9\}$

K. $A \subset C$ and $B \subset D \Leftarrow (A \times B) \subset (C \times D)$

l. $\Leftarrow$ w/ $A, B \neq \emptyset$

$A = \{1\}$

$B = \{6\}$

$C = \{2\}$

$D = \{3\}$