1.1

#1, 3, 4, 5, 6, 7

due Wed 3/9

#2, 8, 9, 10 – Mon π

Cartesian Product

$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$

$\mathbb{R} \times \mathbb{R}$ = 2-dim'l cartesian plane of (x,y) coordinates

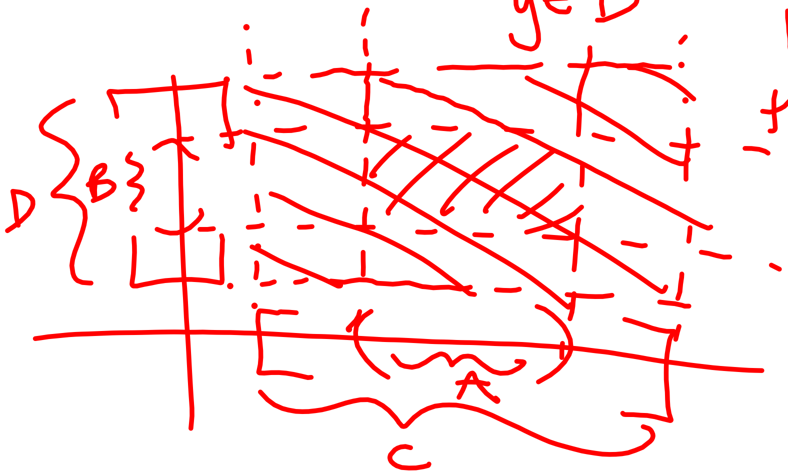
j. $\underbrace{A \subset C}$ and $\underbrace{B \subset D} \Rightarrow (A \times B) \subset (C \times D)$

If $x \in A$, then $x \in C$

If $y \in B$, then $y \in D$

If $(x,y) \in A \times B$, then $(x,y) \in C \times D$

If $x \in A$ and $y \in B$, then $x \in C$ and $y \in D$.



K. $A \subset C$ and $B \subset D \Leftarrow (A \times B) \subset (C \times D)$

false

$A = \{1, 2, 3\}$

$B = \varnothing$

$C = \{4, 5, 6\}$

$D = \{7, 8, 9\}$

k. $A \subset C$ and $B \subset D \Leftarrow (A \times B) \subset (C \times D)$

l. $\Leftarrow$ w/ $A, B \neq \emptyset$

~~$A = \{x\}$~~
~~$B = \{y\}$~~
~~$C = \{y\}$~~
~~$D = \{x\}$~~

~~$(x,y) \neq (y,x)$~~

Nope, it's true!

k. $A \subset C$ and $B \subset D \Leftarrow (A \times B) \subset (C \times D)$

l. $\Leftarrow$ w/ $A, B \neq \emptyset$

Suppose $(A \times B) \subset (C \times D)$, $A, B \neq \emptyset$

We want to show that $A \subset C$ and $B \subset D$.

Let $x \in A$ and $y \in B$.

$\Rightarrow (x,y) \in A \times B$.

Since $(A \times B) \subset (C \times D)$,

$\Rightarrow (x,y) \in C \times D$

$\Rightarrow x \in C$ and $y \in D$.

$\Rightarrow A \subset C$ and $B \subset D$. $\square$

o. $A \times (B - C) = (A \times B) - (A \times C)$

$A \times (B - C) = \{(a, y) \mid a \in A \text{ and } y \in B - C\}$

$y \in B - C \Rightarrow y \in B \text{ and } y \notin C.$

$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

$A \times C = \{(a, c) \mid a \in A \text{ and } c \in C\}$

$(A \times B) - (A \times C) = \{(x, y) \mid (x, y) \in A \times B \text{ and } (x, y) \notin A \times C\}$

o. $A \times (B - C) = (A \times B) - (A \times C)$

$A = \emptyset$, B, C - nice, non-empty ✓

$B = \emptyset$, A, C - nice, nonempty ✓

$C = \emptyset$, A, B - nice, nonempty

o. $A \times (B - C) = (A \times B) - (A \times C)$

$\subseteq$: Let $(x,y) \in A \times (B-C)$.

Want to show that $(x,y) \in (A\times B) - (A \times C)$

$(x,y) \in A \times (B-C) \Rightarrow x \in A$ and $y \in B$ and $y \notin C$

$x \in A$ and $y \in B \Rightarrow (x,y) \in A \times B$

$x \in A$ and $y \notin C \Rightarrow (x,y) \notin A \times C$

$(x,y) \in A \times B$ and $(x,y) \notin A \times C$

$\Rightarrow (x,y) \in (A \times B) - (A \times C)$

o. $A \times (B - C) = (A \times B) - (A \times C)$

$\supseteq$: Let $(x,y) \in (A \times B) - (A \times C)$.

$\Rightarrow (x,y) \in A \times B$ and $(x,y) \notin A \times C$.

$(x,y) \in A \times B \Rightarrow x \in A$ and $y \in B$.

$(x,y) \notin A \times C \Rightarrow x \notin A$ or $y \notin C$.

case 1: $x \in A$ and $y \in B$ and $x \notin A$.

$\Rightarrow A = \emptyset \Rightarrow A \times (B-C) = \emptyset$,

$\Rightarrow (x,y)$ is a non-element of the empty set $A \times (B-C)$

case 2: $x \in A$ and $y \in B$ and $y \notin C$.

$\Rightarrow x \in A$ and $y \in B - C$

$\Rightarrow (x,y) \in A \times (B-C)$. $\square$

P. $(A-B)\times(C-D)=(A\times C-B\times C)-A\times D$

$\subseteq$: Let $(x,y)\in(A-B)\times(C-D)$.

$\Rightarrow \begin{cases} x\in A-B \Rightarrow x\in A \text{ and } x\notin B \\ y\in C-D \Rightarrow y\in C \text{ and } y\notin D. \end{cases}$

$x\in A$ and $y\in C \Rightarrow (x,y)\in A\times C$

$x\notin B \Rightarrow (x,y)\notin B\times C$

$y\notin D \Rightarrow (x,y)\notin A\times D$

$\Rightarrow (x,y)\in(A\times C)-(B\times C)$

$\Rightarrow (x,y)\in[(A\times C)-(B\times C)]-(A\times D)$