1.1

#1, 3, 4, 5, 6, 7

due Wed 3/9

#2, 8, 9, 10 – Mon π

P. $(A-B)\times(C-D) = (A\times C - B\times C) - A\times D$

$\subseteq$: Let $(x,y)\in(A-B)\times(C-D)$.

$\Rightarrow \begin{cases} x\in A-B \Rightarrow x\in A \text{ and } x\notin B \\ y\in C-D \Rightarrow y\in C \text{ and } y\notin D. \end{cases}$

$x\in A$ and $y\in C \Rightarrow (x,y)\in A\times C$

$x\notin B \Rightarrow (x,y)\notin B\times C$

$y\notin D \Rightarrow (x,y)\notin A\times D$

$\Rightarrow (x,y)\in(A\times C)-(B\times C)$

$\Rightarrow (x,y)\in[(A\times C)-(B\times C)]-(A\times D)$

p. $(A-B)\times(C-D) = (A\times C - B\times C) - A\times D$

$\supseteq$: Let $(x,y) \in (A\times C - B\times C) - A\times D$.

$\Rightarrow (x,y) \in (A\times C - B\times C)$ and $(x,y) \notin A\times D$

$(x,y) \in (A\times C - B\times C) \Rightarrow (x,y) \in A\times C$ and $(x,y) \notin B\times C$

$(x,y) \notin A\times D \Rightarrow x \notin A$ or $y \notin D$.

Case 1: $(x,y) \in A\times C$ and $(x,y) \notin B\times C$ and $x \notin A$.

$(x,y) \in A\times C \Rightarrow x \in A$. $x \in A$ and $x \notin A \Rightarrow A = \emptyset$

$\Rightarrow A\times C = \emptyset \Rightarrow (A\times C) - (B\times C) = \emptyset$

$\Rightarrow (A\times C) - (B\times C) - A\times D = \emptyset$

$\emptyset \subseteq (A-B)\times(C-D)$.

Case 2: $(x,y) \in A\times C$ and $(x,y) \notin B\times C$ and $y \notin D$.

$(x,y) \in A\times C \Rightarrow x \in A$ and $y \in C$

$(x,y) \notin B\times C \Rightarrow x \notin B$ or $y \notin C$

Case 2a: $x \in A$ and $y \in C$ and $x \notin B$ and $y \notin D$.

$x \in A$ and $x \notin B \Rightarrow x \in A-B$

$y \in C$ and $y \notin D \Rightarrow y \in C-D$

$x \in A-B$ and $y \in C-D \Rightarrow (x,y) \in (A-B)\times(C-D)$.

Case 2b: $x \in A$ and $y \in C$ and $y \notin C$ and $y \notin D$.

$\Rightarrow C = \emptyset$ (proof is similar to case 1)

7. $F = \{x \mid x \in A \text{ and } \underbrace{(x \in B \Rightarrow x \in C)}_{\text{If } x \in B, \text{ then } x \in C}\}$

$F = A \cap (B \cap C)$

§2 Functions

rule of assignment is a subset r of the cartesian product $C \times D$ of two sets, having the property that each element of C appears as the first coordinate of at most one ordered pair belonging to r.

(how we defined "function" in Precal)

$$(c_1, d_1) = (c_1, d_2) \Rightarrow d_1 = d_2$$

The domain of a rule of assignment $r \subset C \times D$ is the subset of C consisting of all first coordinates of r.

The image set of r is the subset of D consisting of all the second coordinates.

there exists

domain of $r = \{c \in C \mid \exists\, d \in D \text{ s.t. } (c,d) \in r\}$

image of $r = \{d \in D \mid \exists\, c \in C \text{ s.t. } (c,d) \in r\}$

A <u>function</u> f is a rule of assignment r, togther with a set B that contains the image set of r.

the domain of f is the domain of r
the image of f is the image of r
the range of f is the set B.

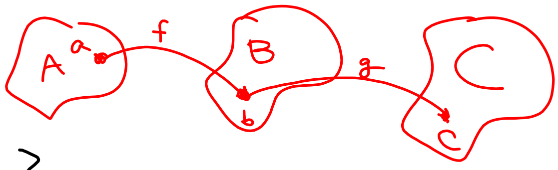
$f: A \longrightarrow B$ f is a function with domain A and range B

$(f: \mathbb{R} \longrightarrow \mathbb{R})$
$f(x) = x^2$

If $f: A \to B$, and $A_0 \subset A$, the <u>restriction</u> of f to A_0 as
$\{(a, f(a)) \mid a \in A_0\}$

$f(x) = \sin x$ $f: \mathbb{R} \to \mathbb{R}$
to construct $f^{-1}(x) = \sin^{-1}(x)$,
we first restrict $f: [-\frac{\pi}{2}, \frac{\pi}{2}] \longrightarrow \mathbb{R}$

Given $f: A \to B$ and $g: B \to C$,
we can define the composition
$f \circ g: A \to C$ as



$$\{(a,c) \mid \text{For some } b \in B,\ f(a) = b \text{ and } g(b) = c.\}$$

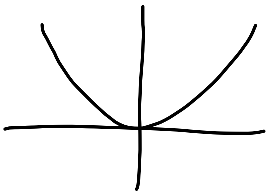
Note that $f \circ g$ is only defined when the range of f equals the domain of g.

$f: A \to B$ is injective (or one-to-one)
if $f(a_1) = f(a_2)$, then $a_1 = a_2$.
(if $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$)

$f: A \to B$ is surjective (or onto)
if for every $b \in B$, there exists $a \in A$ such that $f(a) = b$.

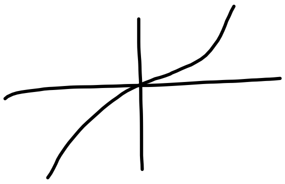
$f : \mathbb{R} \to \mathbb{R}$

$f(x) = x^2$ is <u>not</u> onto



range is not all of $\mathbb{R}$

$f(x) = x^3$ <u>is</u> onto



If f is both one-to-one and onto, we call it <u>bijective</u> and we can define the inverse of $f : A \to B$

$$f^{-1}(B) = \{ a \in A \mid f(a) = b \text{ for some } b \in B \}$$

1.2

#1b $f: A \to B$, $A_0 \subseteq A$; $B_0 \subseteq B$

Show that $f(f^{-1}(B_0)) \subseteq B_0$ and equality holds if f is surjective.

Let $y \in f(f^{-1}(B_0))$.

$\Rightarrow$ y is in the image of $f^{-1}(B_0)$ under f

$\Rightarrow$ $\exists$ $x \in f^{-1}(B_0)$ such that $f(x) = y$.

$\Rightarrow$ x is in the preimage of B_0.

$\Rightarrow$ $\exists$ $y_0 \in B_0$ such that $f(x) = y_0$.

Since f is a function and $f(x) = y$ and $f(x) = y_0$, we must have $y = y_0$.

Hence $y \in B_0$, and $f(f^{-1}(B_0)) \subseteq B_0$.