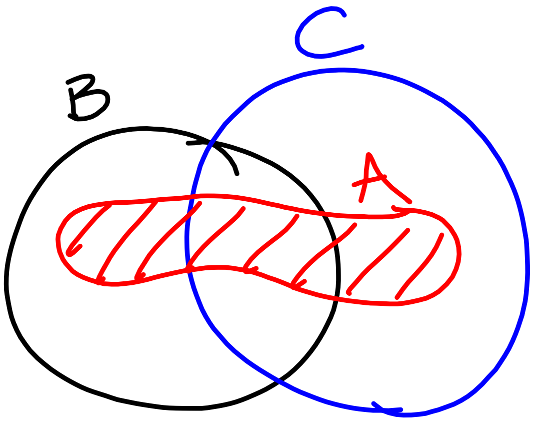


$$x \in \bigcap_{A \in \mathcal{A}} A$$

$$\Rightarrow x \in A \text{ for all } \forall A \in \mathcal{A}$$

$$x \in \bigcup_{A \in \mathcal{A}} A \Rightarrow x \in A \text{ for some (for at least one) } A \in \mathcal{A}$$

§1.2 - Functions

A <u>function</u> f is a rule of assignment (each input maps to one output) together with a set B (range) that contains the image set of the rule.

f is <u>one-to-one</u> or <u>injective</u> if
$f(a) = f(b)$ implies that $a = b$ for all $a, b \in$ domain of f.
If $a \neq b$, then $f(a) \neq f(b)$.

$f: A \to B$ is <u>onto</u> or <u>surjective</u> if for every $b \in B$, there exists $a \in A$ such that $f(a) = b$.

If $f: A \to B$ is both injective & surjective, it is called <u>bijective</u> and has a unique inverse function $f^{-1}: B \to A$

(For a bijective function $f: A \to B$)

If $x \in f^{-1}(B) \Rightarrow f(x) \in B$

$\Rightarrow \exists\, b_0 \in B$ s.t. $f(x) = b_0$ $\quad x = f^{-1}(b_0)$

If $y \in f(A) \Rightarrow f^{-1}(y) \in A$

$\Rightarrow \exists\, a_0 \in A$ s.t. $y = f(a_0)$

s.t. $f^{-1}(y) = a_0$.

$f: A \to B$

$A_0 \subseteq A$, $B_0 \subseteq B$

$f(A_0) = \{ y \in B \mid y = f(a) \text{ for some } a \in A_0 \}$

uniquely defined by def. of function.

$$f^{-1}(B_0) = \{a \in A \mid f(a) \in B_0\}$$

$f^{-1}(y)$ is only uniquely defined if f is injective or one-to-one

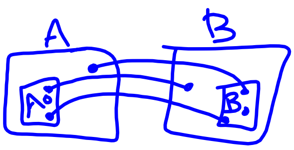
$f: A \to B$; $A_0 \subseteq A$; $B_0 \subseteq B$

The image of A_0 under f

$$f(A_0) = \{b \in B \mid b = f(a) \text{ for some } a \in A_0\}$$

The preimage of B_0 under f

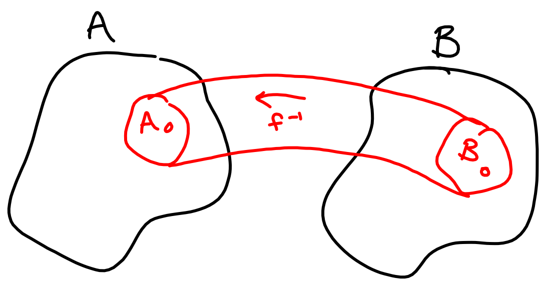
$$f^{-1}(B_0) = \{a \in A \mid f(a) \in B_0\}$$



1. (b) Let $f: A \to B$. $A_0 \subset A$ $B_0 \subset B$

Show that $f(f^{-1}(B_0)) \subset B_0$

and that equality holds if f is onto.



the preimage of B_0 under f is

$A_0 = f^{-1}(B_0) = \{a \mid f(a) \in B_0\}$

$y \in f(f^{-1}(B_0))$

$\Rightarrow \exists\, x \in f^{-1}(B_0)$ s.t. $f(x) = y$

(p19) $\Rightarrow \exists\, b_0 \in B_0$ s.t. $f(x) = b_0$

$y = b_0 \Rightarrow y \in B_0$.

$f(f^{-1}(B_0)) \subseteq B_0$

why don't we have $B_0 = f(f^{-1}(B_0))$ if f is not onto?

$A = \{1, 2, 3, 4, 5\}$

$B = \{6, 7, 8, 9, 10\}$

$B_0 = \{6, 7, 8, 9\}$

$f: A \to B$

$(1, 6)$ $(4, 7)$

$(2, 6)$ $(5, 8)$

$(3, 7)$

$f^{-1}(B_0) = A$

$f(f^{-1}(B_0)) = f(A) = \{6, 7, 8\}$

$f(f^{-1}(B_0)) \subseteq B_0$ BUT

$B_0 \not\subseteq f(f^{-1}(B_0))$

because f is not onto.

$f: A \to B$, onto To prove
$B_0 \subseteq B$ $B_0 \subseteq f(f^{-1}(B_0))$

Let $y \in B_0$. Since f is onto/surjective,
$\exists$ $a_0 \in A$ such that $f(a_0) = y$.
y is the image of a_0 under f.
Since $a_0 \in A$, $f(a_0) \in f(A)$
So $y \in f(A)$.
$f^{-1}(y) \in f^{-1}(B_0)$
$\underbrace{f(f^{-1}(y))} \in f(f^{-1}(B_0))$
$f^{-1}(y) = \{ a_y \in A \mid f(a_y) = y \}$
$f(f^{-1}(y)) = y$.
$\Rightarrow y \in f(f^{-1}(B_0))$.

$f^{-1}(f(x)) = x$ only if f is injective.