2. $f : A \to B$; $A_i \subseteq A$; $B_i \subseteq B$

e. $A_0 \subseteq A_1 \Longrightarrow f(A_0) \subseteq f(A_1)$

Let $y \in f(A_0)$. <y is in the image of A_0>

$\Rightarrow \exists\, q \in A_0$ such that $y = f(q)$.

Since $A_0 \subseteq A_1$ and $q \in A_0$, we have $q \in A_1$.

Since $q \in A_1$, we have $f(q) \in f(A_1)$

But $y = f(q)$, so $y \in f(A_1)$ and hence $f(A_0) \subseteq f(A_1)$.

$f : A \to B$, $A_0, A_1 \subseteq A$

2f. $f(A_0 \cup A_1) \overset{\subseteq}{\underset{\supseteq}{=}} f(A_0) \cup f(A_1)$

$\supseteq$: Let $y \in f(A_0) \cup f(A_1)$

$\Rightarrow y \in f(A_0)$ or $y \in f(A_1)$

Case 1: $y \in f(A_0) \Rightarrow \exists\, m \in A_0$ s.t. $f(m) = y$.

Since $A_0 \subseteq A_0 \cup A_1$, we have $m \in A_0 \cup A_1$

$\Rightarrow f(m) \in f(A_0 \cup A_1)$

Since $y = f(m)$, $y \in f(A_0 \cup A_1)$ and hence $f(A_0) \cup f(A_1) \subseteq f(A_0 \cup A_1)$.

Case 2: $y \in f(A_1)$ <rest similar to case1).

The image of an element is an element of the image of the set containing the element!

If $a \in A$, then $f(a) \in f(A)$

2f. $f(A_0 \cup A_1) \overset{\subseteq}{\underset{\supseteq}{=}} f(A_0) \cup f(A_1)$

$\subseteq$: Let $y \in f(A_0 \cup A_1)$

$\exists\, x \in A_0 \cup A_1$ such that $f(x) = y$

Since $x \in A_0 \cup A_1$, then $x \in A_0$ or $x \in A_1$.

Case 1: $x \in A_0$

$\Rightarrow f(x) \in f(A_0)$

Since $y = f(x) \Rightarrow y \in f(A_0)$

Since $f(A_0) \subseteq f(A_0) \cup f(A_1)$

therefore $y \in f(A_0) \cup f(A_1)$.

And hence, $f(A_0) \cup f(A_1) \supseteq f(A_0 \cup A_1)$

Case 2: $x \in A_1$

proof similar to case 1. ✓ :)

2g

$f(A_0 \cap A_1) \subseteq f(A_0) \cap f(A_1)$

Let $y \in f(A_0 \cap A_1) \Rightarrow \exists\, t \in A_0 \cap A_1$ s.t. $f(t) = y$

$\Rightarrow t \in A_0$ and $t \in A_1$

$\Rightarrow f(t) \in f(A_0)$ and $f(t) \in f(A_1)$

since $f(t) = y \Rightarrow y \in f(A_0)$ and $y \in f(A_1)$

$\Rightarrow y \in f(A_0) \cap f(A_1)$

hence $f(A_0 \cap A_1) \subseteq f(A_0) \cap f(A_1)$

$$\bigcup_{A \in \mathcal{A}} A = A_0 \cup A_1 \cup A_2 \cup \ldots$$

$$\mathcal{A} = \{A_0, A_1, A_2, \ldots\}$$

2g $f(A_0 \cup A_1) \subseteq f(A_0) \cap f(A_1)$

show that equality holds if f is injective.