

$$\bigcap_{A \in \mathcal{A}} A = A_0 \cap A_1 \cap A_2 \cap \dots \cap A_n$$

$x \in \bigcap_{A \in \mathcal{A}} A$ if $x \in A_i$ $\forall i$

$$\bigcup_{A \in \mathcal{A}} A = A_0 \cup A_1 \cup A_2 \cup \dots$$

$x \in \bigcup_{A \in \mathcal{A}} A_i$ if $x \in A_i$ for at least one $A_i \in \mathcal{A}$.

$$f\left(\bigcap_{A \in \mathcal{A}} A\right) \subseteq \bigcap_{A \in \mathcal{A}} f(A)$$

\subseteq : Let $y \in f\left(\bigcap_{A \in \mathcal{A}} A\right)$

$$\Rightarrow f^{-1}(y) \in \left(\bigcap_{A \in \mathcal{A}} A\right)$$

$$\Rightarrow f^{-1}(y) \in A \text{ for every } A \in \mathcal{A}$$

$$\Rightarrow y \in f(A) \text{ for every } A \in \mathcal{A}$$

$$\Rightarrow y \in \bigcap_{A \in \mathcal{A}} f(A)$$

$$\text{hence } f\left(\bigcap_{A \in \mathcal{A}} A\right) \subseteq \bigcap_{A \in \mathcal{A}} f(A)$$

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Suppose, in addition, that f is injective (one-to-one).

To show that $f\left(\bigcap_{A \in \mathcal{A}} A\right) \supseteq \bigcap_{A \in \mathcal{A}} f(A)$

\supseteq : Let $y \in \bigcap_{A \in \mathcal{A}} f(A)$

$\Rightarrow y \in f(A)$ for every $A \in \mathcal{A}$

$\Rightarrow f^{-1}(y) \in A$ for every $A \in \mathcal{A}$

$\Rightarrow \exists x_0 \in A_0, x_1 \in A_1, x_2 \in A_2, \dots, x_i \in A_i$

such that $f(x_i) = y$.

Since the function is injective,

$f(x_i) = f(x_j) \Rightarrow x_i = x_j = x_0$

$\Rightarrow x_0 \in A$ for every $A \in \mathcal{A}$

$\Rightarrow x_0 \in \bigcap_{A \in \mathcal{A}} A$

$\Rightarrow f(x_0) \in f\left(\bigcap_{A \in \mathcal{A}} A\right)$

Since $y = f(x_0) \Rightarrow y \in f\left(\bigcap_{A \in \mathcal{A}} A\right)$

Hence

$$f\left(\bigcap_{A \in \mathcal{A}} A\right) \supseteq \bigcap_{A \in \mathcal{A}} f(A)$$

1.2

Given $f: A \rightarrow B$ and $g: B \rightarrow C$,

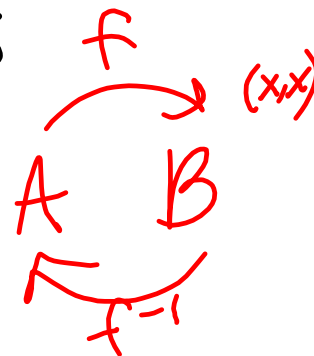
$$A \xrightarrow{f} B \xrightarrow{g} C$$

$g \circ f: A \rightarrow C$ has the rule

$\star \left\{ (a, c) \mid \text{For some } b \in B, f(a) = b \text{ and } g(b) = c \right\}$

Lemma: If a function has an inverse, then it is bijective.

f & g are inverses if $(f \circ g)(x) = x$ & $(g \circ f)(x) = x$



$$\frac{1.2}{4}. f: A \rightarrow B, g: B \rightarrow C$$

(a) If $C_0 \subseteq C$, show that

$$(g \circ f)^{-1}(C_0) = f^{-1}(g^{-1}(C_0)).$$

\subseteq : $\left[\text{Let } h(x) = g(f(x)) \text{ } h: A \rightarrow C \right]$ If $a_0 \in h^{-1}(C_0) \Rightarrow h(a_0) \in C_0$

Let $a_0 \in (g \circ f)^{-1}(C_0)$

$\Rightarrow (g \circ f)(a_0) \in C_0$

$\Rightarrow \exists c_0 \in C_0$ such that $(g \circ f)(a_0) = c_0$

$\Rightarrow \exists b_0 \in B$ such that $f(a_0) = b_0$ and $g(b_0) = c_0$ since f is surjective

$\Rightarrow b_0 = g^{-1}(c_0) \Rightarrow b_0 \in g^{-1}(C_0)$

$f(a_0) = b_0 \Rightarrow a_0 = f^{-1}(b_0)$

$b_0 \in g^{-1}(C_0) \Rightarrow f^{-1}(b_0) \in f^{-1}(g^{-1}(C_0))$

Since $a_0 = f^{-1}(b_0)$, $a_0 \in f^{-1}(g^{-1}(C_0))$

and hence $(g \circ f)^{-1}(C_0) \subseteq f^{-1}(g^{-1}(C_0))$.

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(a) If $C_0 \subseteq C$, show that

$$(g \circ f)^{-1}(C_0) = f^{-1}(g^{-1}(C_0)).$$

\supseteq :

1.2

$\neq 1a$

$2a, b, c, d$

Monday, 3/21