1.2

\# 1a

2a, b, c, d

Monday, 3/21

1.2

Given $f: A \to B$ and $g: B \to C$, $A \xrightarrow{f} B \xrightarrow{g} C$

$g \circ f : A \to C$ has the rule

☆ $\{(a,c) \mid \underline{\text{For some } b \in B,\ f(a) = b \text{ and } g(b) = c}\}$

Lemma: If a function has an inverse, then it is bijective.

f & g are inverses if
$(f \circ g)(x) = x$ & $(g \circ f)(x) = x$

$$A \underset{f^{-1}}{\overset{f}{\rightleftarrows}} B \quad (x,x)$$

1.2
4. $f: A \to B$, $g: B \to C$

(a) If $C_0 \subseteq C$, show that
$(g \circ f)^{-1}(C_0) = f^{-1}(g^{-1}(C_0))$.

$\subseteq$: $\boxed{\text{Let } h(x) = g(f(x)) \quad h: A \to C}$ If $a_0 \in h^{-1}(C_0)$ $\Rightarrow h(a_0) \in C_0$

Let $a_0 \in (g \circ f)^{-1}(C_0)$

$\Rightarrow (g \circ f)(a_0) \in C_0$

$\Rightarrow \exists\ c_0 \in C_0$ such that $(g \circ f)(a_0) = c_0$ since f is surjective,

$\Rightarrow \exists\ b_0 \in B$ such that $f(a_0) = b_0$ and $g(b_0) = c_0$.

$\Rightarrow b_0 = g^{-1}(c_0) \Rightarrow b_0 \in g^{-1}(C_0)$

$f(a_0) = b_0 \Rightarrow a_0 = f^{-1}(b_0)$

$b_0 \in g^{-1}(C_0) \Rightarrow f^{-1}(b_0) \in f^{-1}(g^{-1}(C_0))$

Since $a_0 = f^{-1}(b_0)$, $a_0 \in f^{-1}(g^{-1}(C_0))$

and hence $(g \circ f)^{-1}(C_0) \subseteq f^{-1}(g^{-1}(C_0))$.

1.2
4. $f: A \to B$, $g: B \to C$

(a) If $C_0 \subseteq C$, show that
$(g \circ f)^{-1}(C_0) = f^{-1}(g^{-1}(C_0))$.

$\supseteq$: let $y \in f^{-1}(g^{-1}(C_0))$.

$f(y) \in g^{-1}(C_0)$.

$g(f(y)) \in C_0$.

$(g \circ f)(y) \in C_0$.

$y \in (g \circ f)^{-1}(C_0)$.

hence, $f^{-1}(g^{-1}(C_0)) \subseteq (g \circ f)^{-1}(C_0)$.

2e. $A_0 \subset A_1 \Rightarrow f(A_0) \subset f(A_1)$

Suppose $A_0 \subset A_1$.

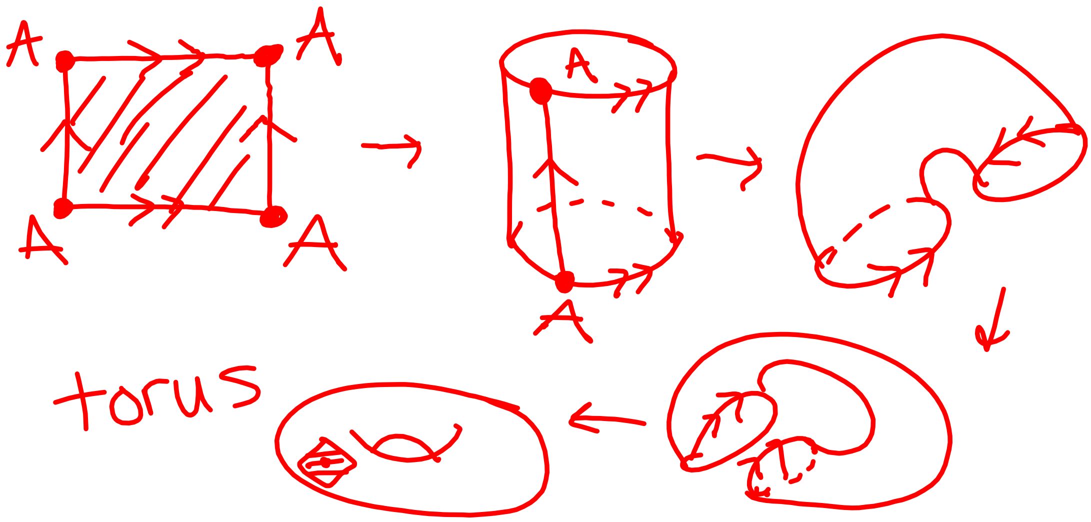
Let $y \in f(A_0)$.

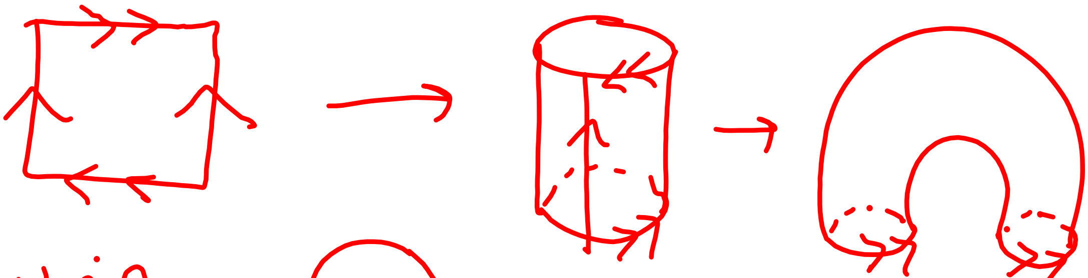
$f^{-1}(y) \in A_0$.

Since $A_0 \subset A_1$ then $f^{-1}(y) \in A_1$

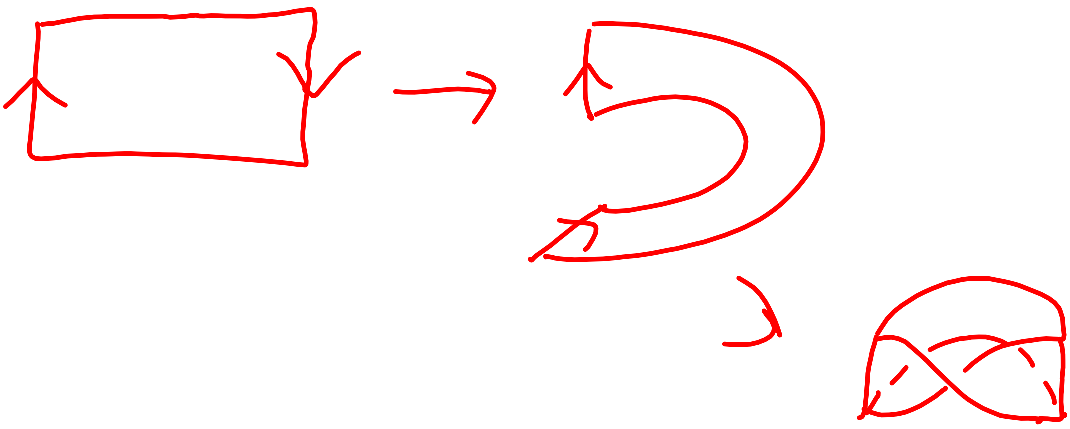
$\Rightarrow y \in f(A_1)$

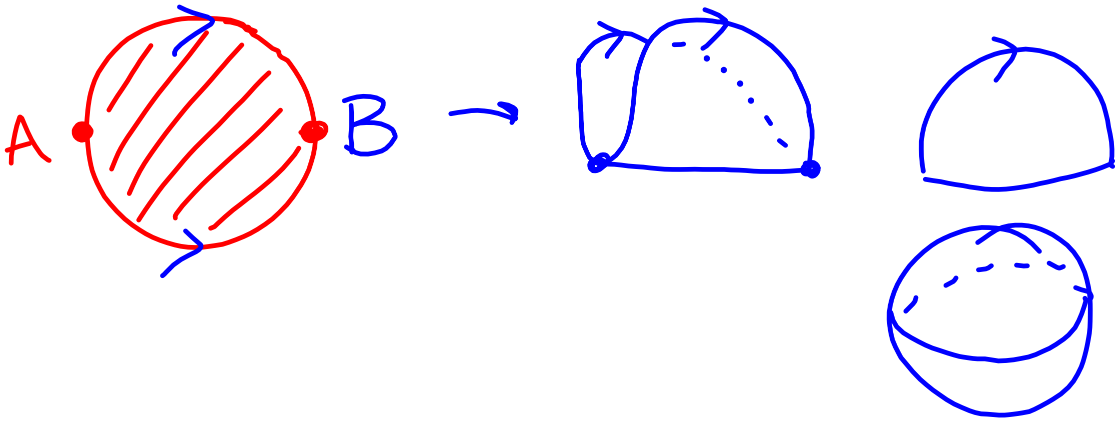
Hence, $f(A_0) \subset f(A_1)$.



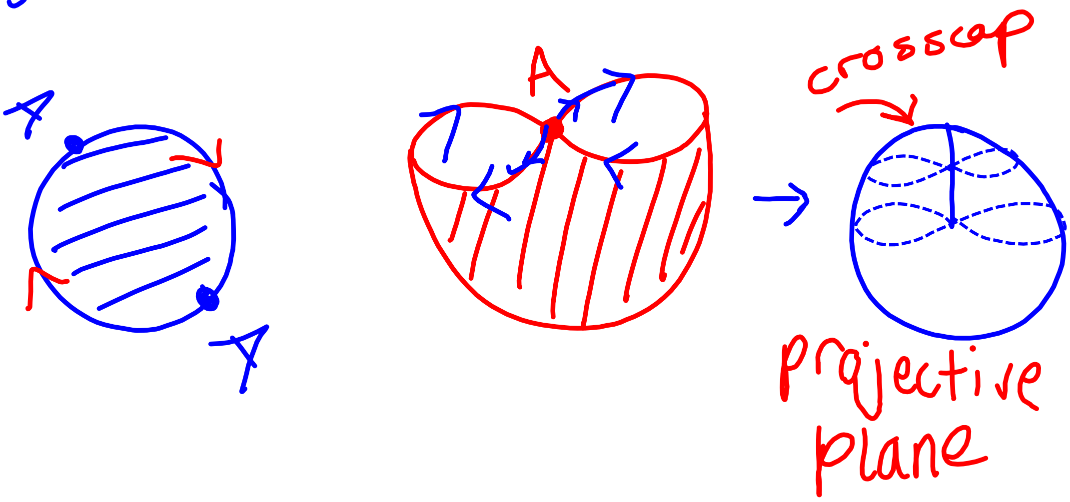


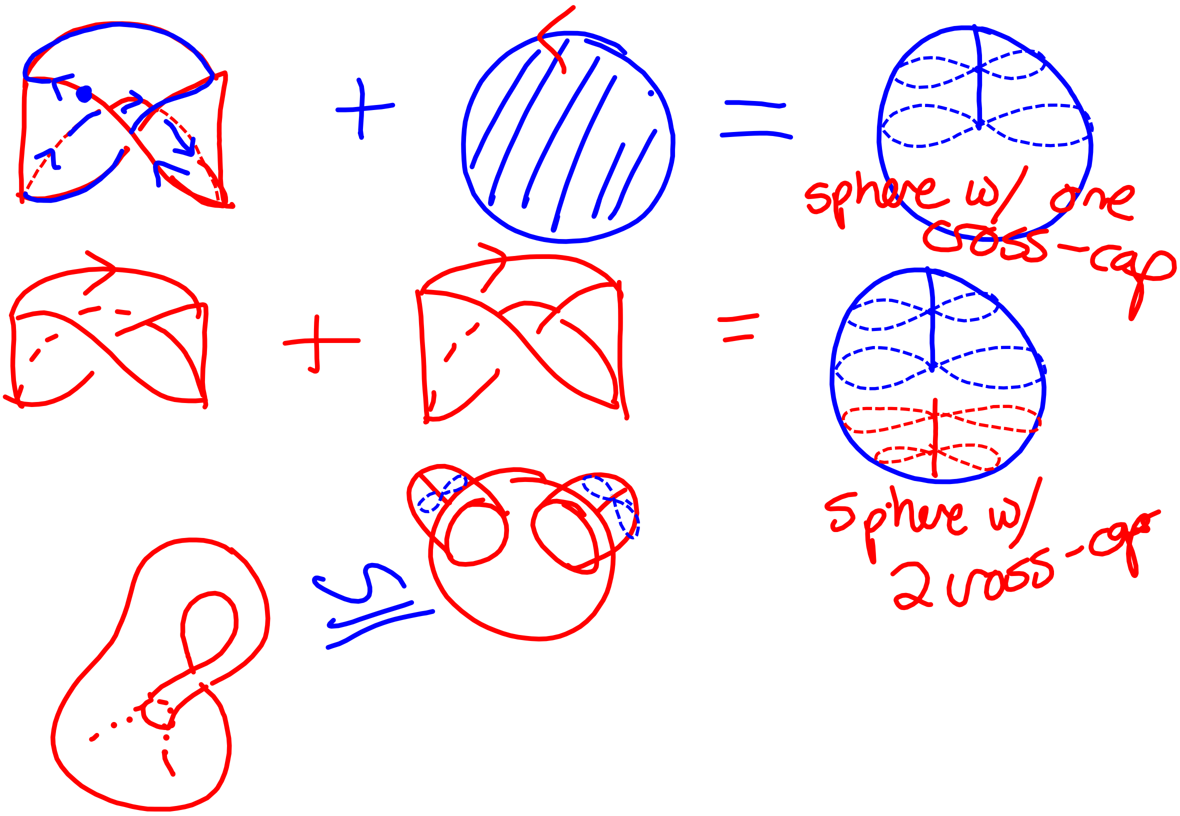
Klein bottle

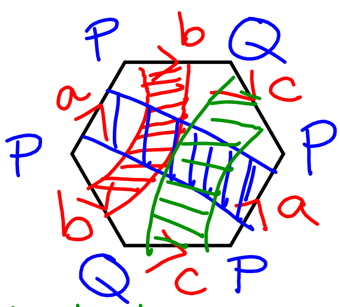



A
B

disk/disc


A
A
A
crosscap
projective plane


sphere w/ one cross-cap
sphere w/ 2 cross-cap



orientable surface w/ no boundary

$abca^{-1}c^{-1}b^{-1}$

Euler characteristic

$\chi(S) = \#V - \#E + \#F$

$V=2$

$E=3$

$F=1$

$\chi(S) = 2-3+1$

$= 0$

Genus of an orientable surface

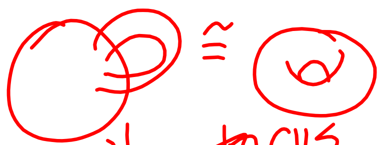
$\chi(S) = 2-2g+b$

$0 = 2-2g+0$

$2g = 2$

$\boxed{g=1}$

# of handles



sphere w/ one handle

torus