If $x<0$, $y<0$ $\quad y<0$, $-y>0$ $\quad x<0$, $-x>0$

$$|x||y| = (-x)(-y) = xy \qquad (-x)(-y) > 0\cdot(-y)$$

$$xy>0$$

$$|xy| = \begin{cases} xy & \text{if } xy>0 \\ -xy & \text{if } xy<0 \end{cases}$$

$$|a+b| \le |a| + |b| \quad \forall\, a,b \in \mathbb{R}$$

10. $\big||x|-|y|\big| \le |x-y|$

$$\big||x|-|y|\big| = \begin{cases} |x|-|y|, & |x|>|y| \\ |y|-|x|, & |y|>|x| \end{cases}$$

$$|x|-|y| = |x-y+y| - |y| \le |x-y| + |y| - |y| = |x-y|$$

$$|y|-|x| = |y-x+x| - |x| \le |y-x| + |x| - |x| = |x-y|$$

8. $2xy \leq x^2+y^2$

$x^2+y^2 \geq 2xy$

$x^2-2xy+y^2 \geq 0$

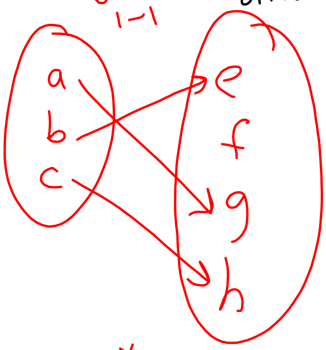
$(x-y)^2 \geq 0$

$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad-bc$

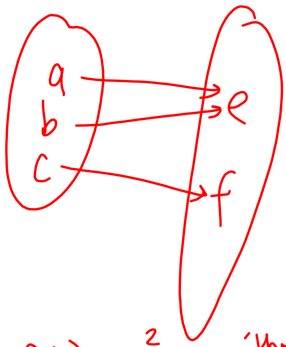
$\mathbb{R} = \{x \mid x\in\mathbb{R}\}$

$\mathbb{R}^2 = \{(x,y) \mid x,y\in\mathbb{R}\}$

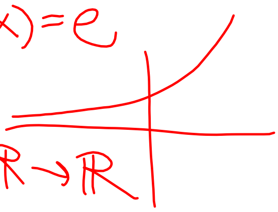
surjective 1-1 but not onto




onto/ injective but not 1-1



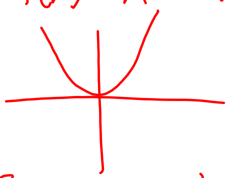

$f(x)=e^x$



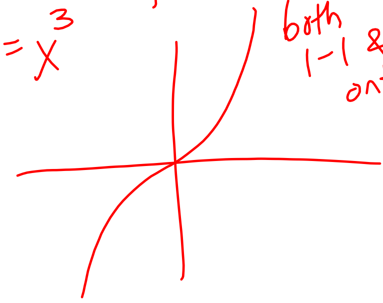
$f:\mathbb{R}\to\mathbb{R}$

$f(x)=x^2$ neither 1-1 or onto

$f:\mathbb{R}\to\mathbb{R}$



$f(x)=x^3$ both 1-1 & onto



$f:A\to B$ is <u>injective</u> (one-to-one)

If for all $x,y\in A$ we have

$f(x)=f(y)$ implies $x=y$

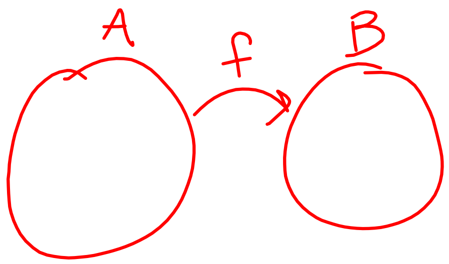
contrapositive $(a\to b\ ,\ \sim b\to\sim a)$

If $x\neq y$, then $f(x)\neq f(y)$.

$f: A \to B$ is <u>surjective</u> (or onto)

if $\forall\, b \in B$, $\exists\, a \in A$ such that $f(a) = b$.

If $f$ is both 1-1 & onto it is called a 1-1 correspondence




If $f: A \to B$ is one-to-one, then for $b \in B$, we define the inverse $f^{-1}(b)$ to be the unique $a \in A$ such that $f(a) = b$.

$A = \{x, y, z\}$ $B = \{a, b, c\}$ $C = \{x, a\}$

$A \cup B = \{x, y, z, a, b, c\}$

$A \cap B = \emptyset$ "empty set"

$A \cap C = \{x\}$

$A \setminus C = A - C = \{y, z\}$

$A \cup \emptyset = A$

$A \cap \emptyset = \emptyset$

$A - \emptyset = A$

1.2

#1 a-e

#2 a, c

#4

#10 b, c

Read 1.3

If $A \subseteq B$,

$x \in A \Rightarrow x \in B$

3. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

case 1 $\subseteq$ : Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$. $x \in B \cup C \Rightarrow x \in B$ or $x \in C$.

case 1a: $x \in A$ and $x \in B \Rightarrow x \in A \cap B$

$A \cap B \subseteq (A \cap B) \cup (A \cap C)$ $\Rightarrow x \in (A \cap B) \cup (A \cap C)$

case 1b: $x \in A$ and $x \in C \Rightarrow x \in A \cap C$

$A \cap C \subseteq (A \cap B) \cup (A \cap C)$

case 2 $\supseteq$ Let $x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in A \cap B$ or $x \in A \cap C$

$x \in A - B \Rightarrow x \in A$ and $x \notin B$.

$x \notin (A \cap B) \Rightarrow x \notin A$ or $x \notin B$

$x \notin (A \cup B) \Rightarrow x \notin A$ and $x \notin B$