A set S is countable iff $\exists f: \mathbb{N} \to S$ one-to-one & onto.

Lemma: Finite sets are countable.

Pf: a set S is finite iff $\exists$ one-to-one & onto function $g: \{1, 2, \ldots, n\} \to S$ for some finite n. $\{1, 2, \ldots, n\} \subseteq \mathbb{N}$, hence this S is countable.

Thm: The union of a finite set & a countable set is countable.

Pf: Let F be a finite set & C be a countable set. To show $F \cup C$ is countable.

$F = \{a_1, a_2, \ldots, a_n\}$

$C = \{b_1, b_2, \ldots\}$

$g: \mathbb{N} \to F \cup C$ defined by

$g(1) = a_1$ $\quad g(n+1) = b_1$

$g(2) = a_2$ $\quad g(n+2) = b_2$,

$\ldots$ $\quad \ldots$

$g(n) = a_n$

is a 1-1 & onto function from $\mathbb{N}$ to $F \cup C$

$A = \{a_1, \ldots, a_n, \ldots\}$ $B = \{b_1, \ldots, b_n, \ldots\}$

both countable

$f: \mathbb{N} \to A \cup B$

$f(1) = a_1$ $\quad f(4) = b_2$

$f(2) = b_1$ $\quad \ldots$

$f(3) = a_2$

$f(2n-1) = a_n$

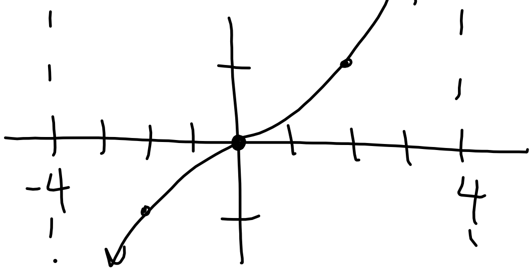
$f(2n) = b_n$

$$f(x) = \cancel{a}\cdot \tan(\overset{\pi/8}{b}x + c) \cancel{+d}$$

$$f:(-1,7)\to\mathbb{R}$$

period for $\tan x = \frac{\pi}{b} = 8 \Rightarrow b = \frac{\pi}{8}$

$\tan \frac{\pi}{8}x$



need phase shift of $+3$

$-\frac{c}{b} = 3 = \frac{-c}{\pi/8}$

$\Rightarrow c = -3\pi/8$

$$f(x) = \tan\left(\tfrac{\pi}{8}x - \tfrac{3\pi}{8}\right)$$

is 1-1, onto $f:(-1,7)\to\mathbb{R}$

$f:A\to B$ $\quad$ A, B finte

$\{(a,b)\mid a\in A,\ b=f(a)\in B\}$

$A=\{a\},\ B=\{b\}$ $\quad\{(a,b)\}$

$A=\{a_1,a_2\},\ B=\{b\}$ $\quad\{(a_1,b),(a_2,b)\}$

$A=\{a_1,a_2\},\ B=\{b_1,b_2\}$ $\quad\{(a_1,b_1)(a_2,b_2)\}$ or $\{(a_2,b_1),(a_1,b_2)\}$ $\{(a_1,b_1),(a_2,b_1)\}$ or $\{(a_1,b_2),(a_2,b_2)\}$ or

$A=\{a_1,a_2,a_3\},\ B=\{b_1,b_2,b_3\}$

$\to b_1$ $\{1,2\to1,3\to2\}$, $\{1,2\to1,3\to3\}$

$\to b_2$

b_3

If A has m elements & B has n elements,

$f:A\to B$ has up to

$\{(a_1,b_1),(a_2,b_1),(a_3,b_1)\}$ $\{a_1, b_1 \quad a_2 b_2 \quad a_3 b_1\}$

$\{ \quad b_2 \quad b_2 \quad b_2 \}$ $\{ \quad b_2\}$

$\{ \quad b_3 \quad b_3 \quad b_3 \}$ $\{ \quad b_3$

$\{(a_1,b_1),(a_2,b_1),(a_3,b_2)\}$ $\{ \quad b_1 \quad b_3 \quad b_1\}$

$\{ \quad b_3\}$ $\{ \quad b_2\}$

$\{ \quad b_2 \quad b_2 \quad b_1\}$ $\{ \quad b_3\}$

$\{ \quad b_3\}$

$\{ \quad b_3 \quad b_3 \quad b_1\}$

$\{ \quad b_3 \quad b_3 \quad b_2\}$

a1	a2	a3		a1	a2	a3		a1	a2
b1	b1	b1		b1	b1	b1		b1	b1
b1	b1	b2			b1	b2		b1	b2
b1	b1	b3			b2	b1		b1	b3
b1	b2	b1			b2	b2		b2	b1
b1	b2	b2		b2	b1	b1		b2	b2
b1	b2	b3			b1	b2		b2	b3
b1	b3	b1			b2	b1		b3	b1
b1	b3	b2			b2	b2		b3	b2
b1	b3	b3						b3	b3
b2	b1	b1		f:M=3->N=2					
b2	b1	b2						f:M=2->N=3	
b2	b1	b3		cardinality N^M					
b2	b2	b1							
b2	b2	b2							
b2	b2	b3							
b2	b3	b1							
b2	b3	b2							
b2	b3	b3							
b3	b1	b1							
b3	b1	b2							
b3	b1	b3							
b3	b2	b1							
b3	b2	b2							
b3	b2	b3							
b3	b3	b1							
b3	b3	b2							
b3	b3	b3							

$$f : \mathbb{N} \to \{a, b\}$$

$$F_1 = \{(1, x_1), (2, x_2), (3, x_3), \ldots\}$$

$$F_2 = \{(1, y_1),$$

1.3

#9 d

1.4

#3, 5, 6