$$a_n = 3 + 2^{-n} \longrightarrow 3$$

$$|3 + 2^{-n} - 3| \overset{\text{want}}{<} \varepsilon$$

$$|2^{-n}| < \varepsilon$$

$$\frac{1}{2^n} < \varepsilon$$

$$\log_2 \frac{1}{2^n} < \log_2 \varepsilon$$

$$\log_2 1 - \log_2 2^n < \log_2 \varepsilon$$

$$-n < \log_2 \varepsilon$$

$$n > \boxed{-\log_2 \varepsilon = N}$$

$$0 < \frac{1}{2^n} < 1$$

$$\log_a\left(\frac{1}{2^n}\right) < 0$$

$$a_n = \sqrt{\frac{n}{n+1}} \to 1$$

want $\left|\sqrt{\frac{n}{n+1}} - 1\right| < \varepsilon$

$$1 - \sqrt{\frac{n}{n+1}} < \varepsilon$$

$$1 - \varepsilon < \sqrt{\frac{n}{n+1}}$$

$$\sqrt{\frac{n}{n+1}} > 1 - \varepsilon$$

$$\frac{n}{n+1} > (1-\varepsilon)^2$$

$$\frac{n+1}{n} < \frac{1}{(1-\varepsilon)^2}$$

$$\frac{n}{n} + \frac{1}{n}$$

$$1 + \frac{1}{n} < \frac{1}{(1-\varepsilon)^2}$$

$$\frac{1}{n} < \frac{1}{(1-\varepsilon)^2} - 1$$

$$n > \frac{1}{\frac{1}{(1-\varepsilon)^2} - 1}$$

$$n > \frac{(1-\varepsilon)^2}{1 - (1-\varepsilon)^2}$$

$$a_n = \frac{3n+1}{n+2} \to 3$$

we want $\left|\frac{3n+1}{n+2} - 3\right| < \varepsilon$

$$\left|\frac{3n+1-3n-6}{n+2}\right| < \varepsilon$$

$$\left|\frac{-5}{n+2}\right| < \varepsilon$$

$$\frac{5}{n+2} < \varepsilon$$

$$\frac{n+2}{5} > \frac{1}{\varepsilon}$$

$$n+2 > 5/\varepsilon$$

$$n > 5/\varepsilon - 2$$

$$\frac{2}{\ln n} < \varepsilon$$

$$\frac{\ln n}{2} > \frac{1}{\varepsilon}$$

$$\ln n > \frac{2}{\varepsilon}$$

$$e^{\ln n} > e^{2/\varepsilon}$$

$$n > e^{2/\varepsilon}$$

$$e^{2/\varepsilon} \geq 2$$

$$2/\varepsilon \geq \ln 2$$

$$\frac{\varepsilon}{2} \leq \frac{1}{\ln 2}$$

$$\varepsilon \leq \frac{2}{\ln 2}$$

Take $\frac{2}{\ln 2} \geq \varepsilon > 0$

To prove 2^n diverges.

i.e. to find an N so that for some M $2^n \geq M \ \forall n \geq N$

Let $M>0$ be given.

want $2^N \geq M$

take $N = \log_2 M$

4.c $a_n = \sqrt{\ln n}$

$\sqrt{\ln N} \geq M$

find N

$\ln N \geq M^2$

$N \geq e^{M^2}$

4.d $a_n = \frac{n!}{2^n}$

$\frac{N!}{2^N} \geq M$

$N! \geq M2$

$\frac{N!}{2^N} \geq \frac{1}{2^N} \quad \forall N$

$\frac{1}{2^N} \geq M$

$2^{-N} \geq M$

$\log_2 2^{-N} \geq \log_2 M$

$-N \log_2 2 \geq \log_2 M$

$N \leq -\log_2 M$

$$\frac{n!}{2^n}$$

suppose true for

$$\frac{N!}{2^N} \geq M$$

$$\frac{1}{2^1}, \frac{2\cdot 1}{2^2}, \frac{3\,2\cdot 1}{2^3}, \frac{4\,3\,2}{2^4}$$

$$\frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{3}{2},$$

$$\frac{(N+1)!}{2^{N+1}} = \frac{(N+1)N!}{2\cdot 2^N} \geq \frac{N+1}{2}\cdot M \geq M$$